Acquisition of Composite GNSS Signals

D. Borio, C. O'Driscoll and G. Lachapelle

Position, Location And Navigation (PLAN) Group
Department of Geomatics Engineering
Schulich School of Engineering
University of Calgary

ION Alberta Section Meeting
17 April 2008

Outline

- Composite GNSS signals
- Single code period acquisition
  - non-coherent channel combining
  - coherent channel combining with sign recovery
  - differentially coherent channel combining
- Multiple code period acquisition
  - without sign recovery
  - with sign recovery
- Real data analysis
- Conclusions

Composite GNSS signals

Principle:

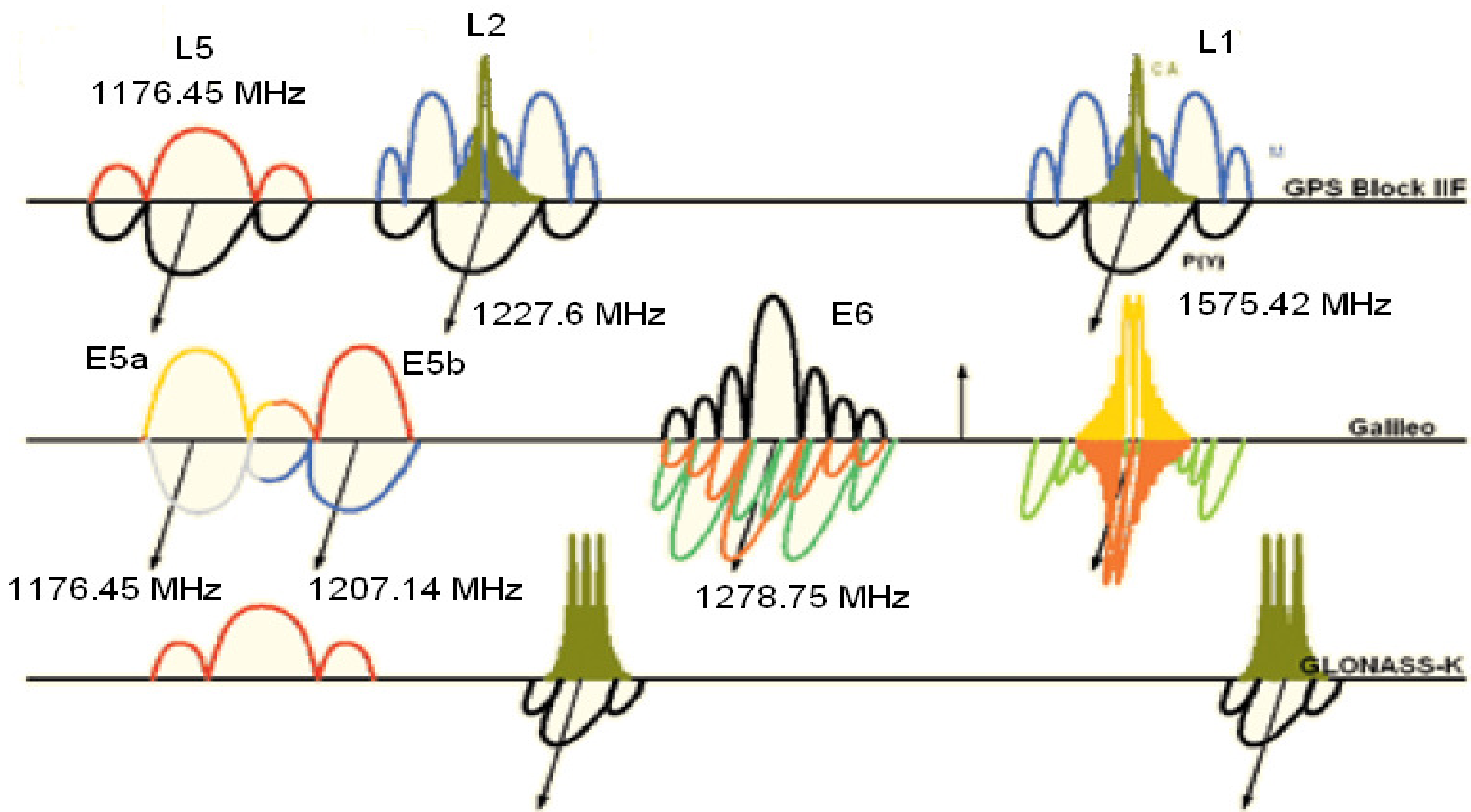
two different pieces of information are required for determining the user position → **Satellite position** / **Pseudo-range (and carrier phase)**

The transmission of the two messages on a single channel can be troublesome

Solution:

split the two messages on two different channels → **Data channel** / **Pilot channel**

Composite GNSS signals: some examples



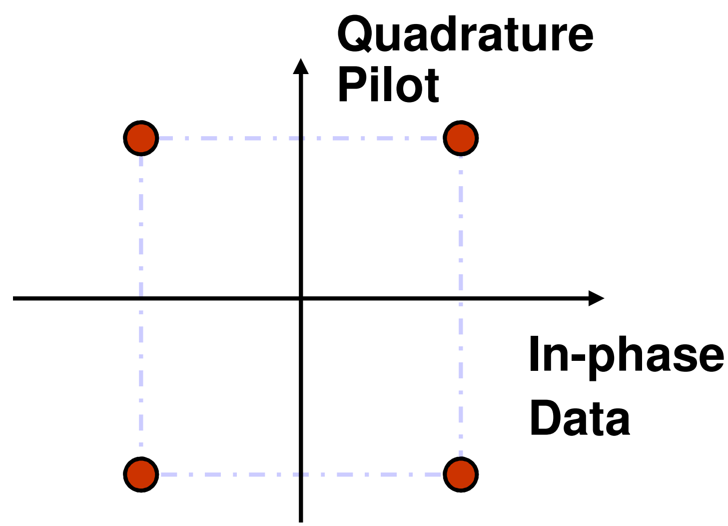
Galileo: **all the new Galileo signals have a data/pilot structure**

GPS: **L5, L2C and L1C signals**

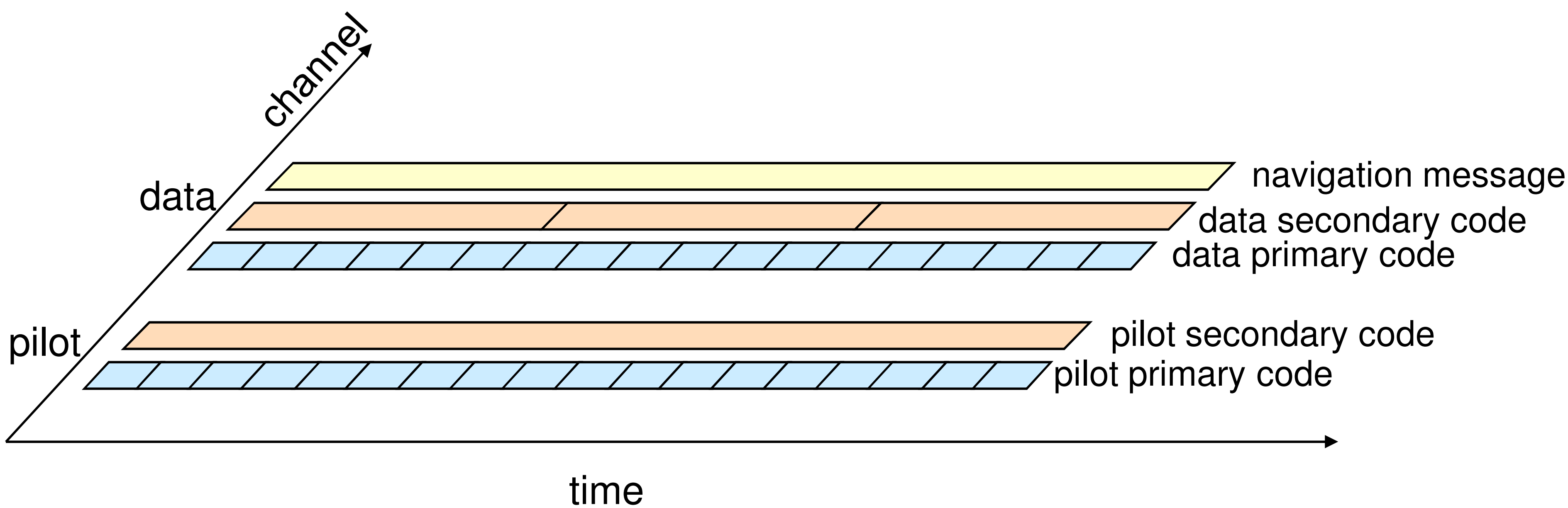
From S. Lo, A. Chen, P. Enge, G. Gao, D. Akos, J. Issler, L. Ries, T. Grelier and J. Dantepal, “GNSS Album Image and Spectral Signatures of the New GNSS Signals”, Inside GNSS May/June 2006

QPSK signals

Data and pilot channels are transmitted with a 90 degree phase difference on the in-phase and quadrature branches



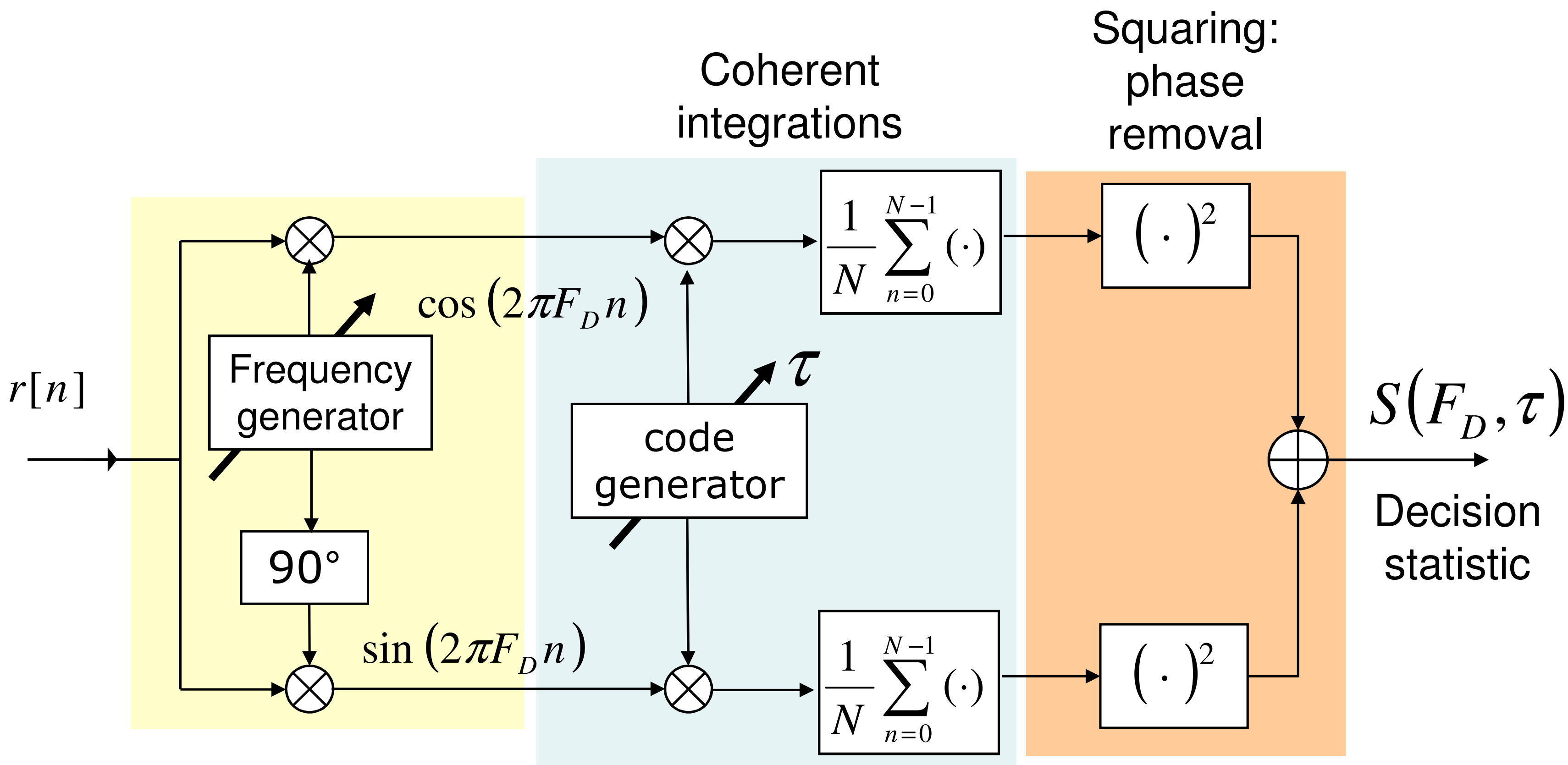
Signal structure



Single Code Period Acquisition

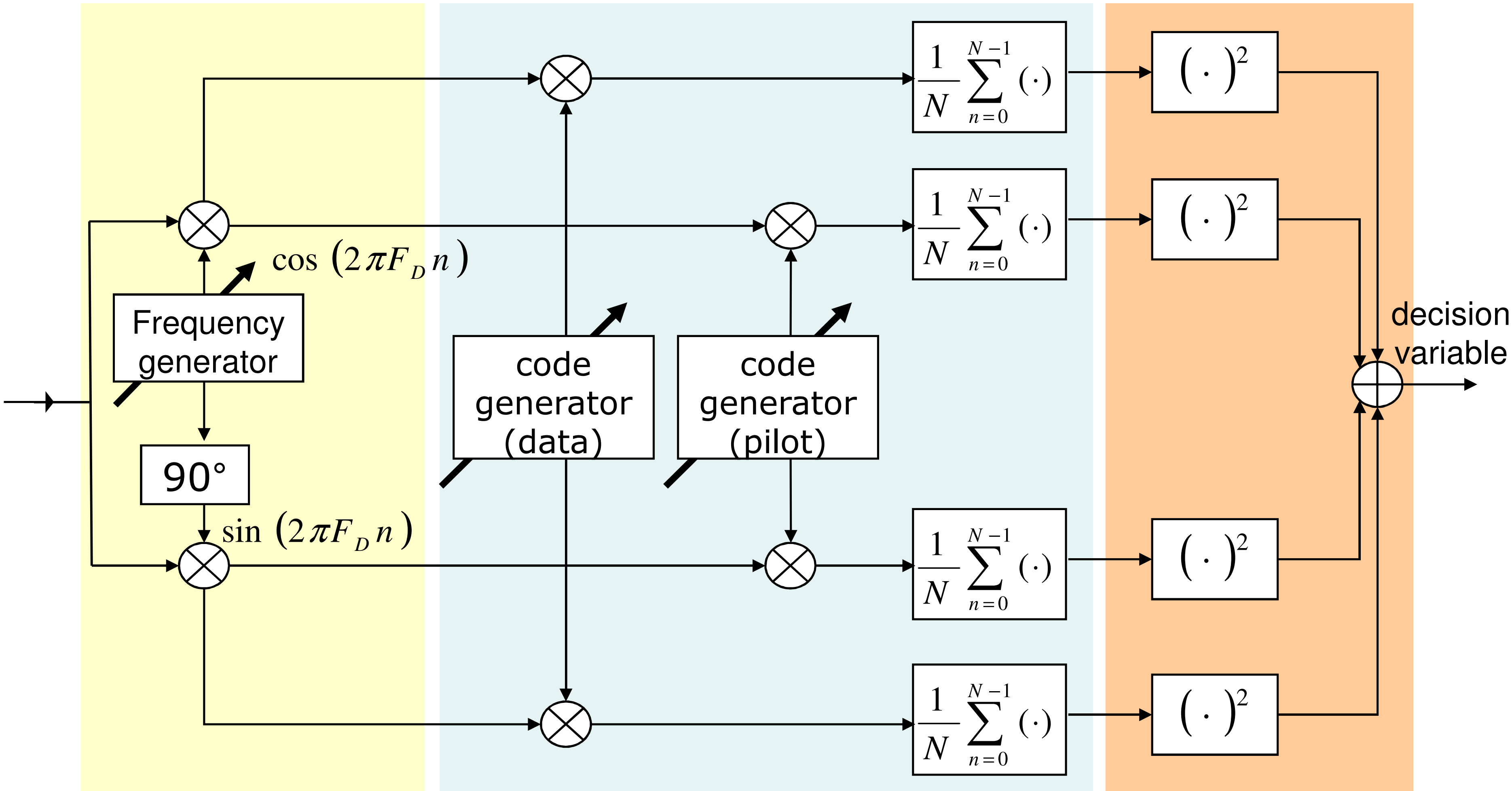
- Single channel acquisition
- Non-coherent channel combining
- Coherent channel combining with sign recovery
- Differentially coherent channel combining

Single Channel Acquisition



- ✓ **Only half of the available power is employed;**
- ✓ **It requires the lowest computational load.**
- ✓ **It works as a traditional acquisition block for BPSK signals.**

Non-coherent channel combining



A. J. V. Dierendonck and J. J. Spilker Jr., “Proposed civil GPS signal at 1176.45 MHz: In-phase/quadrature codes at 10.23 MHz chip rate,” in Proc. of ION Annual Meeting (AM), Cambridge, MA, June 1999, pp. 761 – 770.

F. Bastide, O. Julien, C. Macabiau, and B. Roturier, “Analisis of L5/E5 acquisition, tracking and data demodulation thresholds,” in Proc. of ION GPS/GNSS, Portland, OR, Sept. 2002, pp. 2196 – 2207

Coherent channel combining with sign recovery

Principle

If the sign between data and pilot were known all the signal power could be recovered by employing the correct composite code:

$$c^{+}[n] = c_I[n] + jc_Q[n]$$
$$c^{-}[n] = c_I[n] - jc_Q[n]$$

The sign between data and pilot is unknown → **Parameter to be estimated**

$$S(F_D,\tau) = \max\left\{\left|S^{+}(F_D,\tau)\right|^2, \left|S^{-}(F_D,\tau)\right|^2\right\}$$

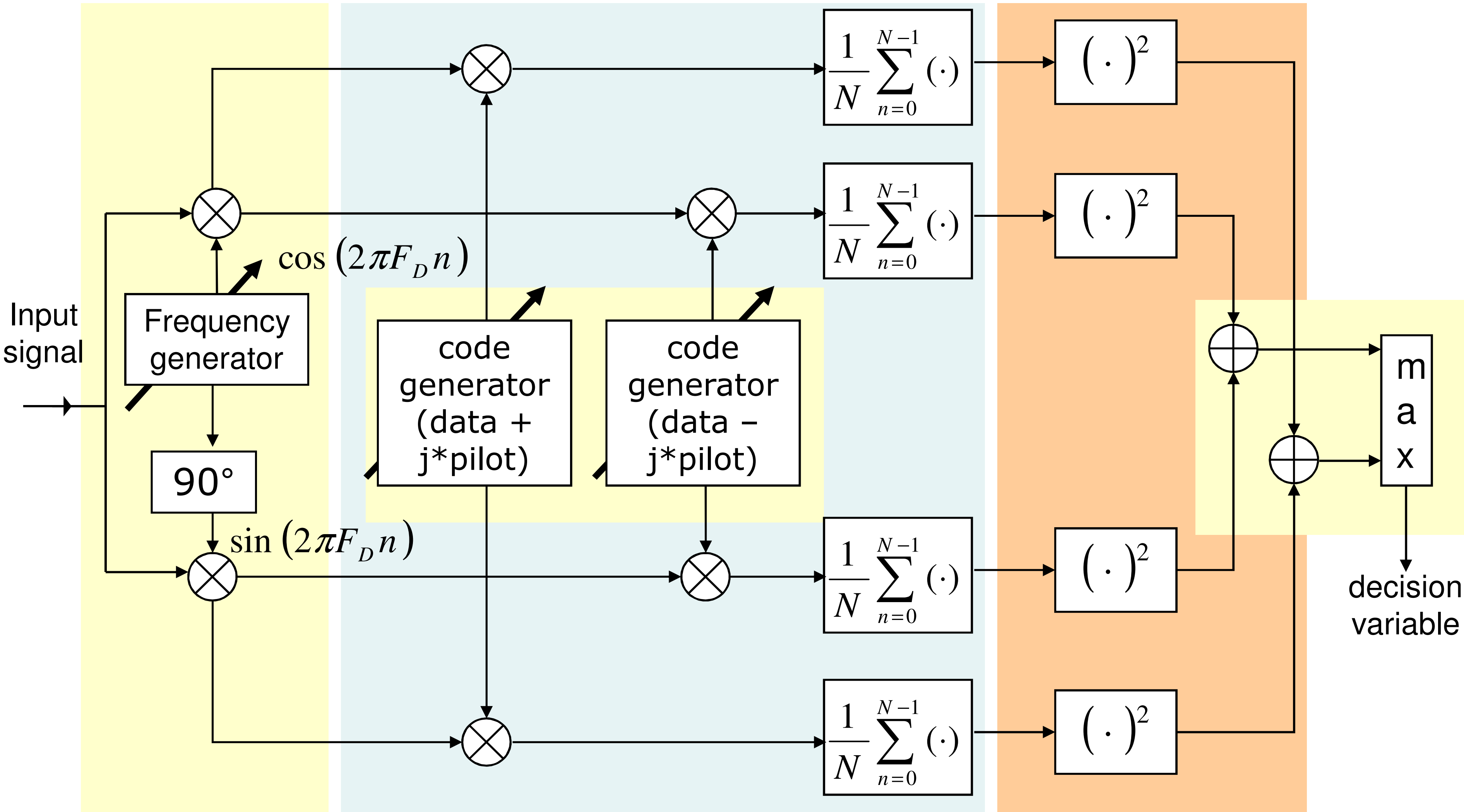
Correlation with the code + (→ $\left|S^{+}(F_D,\tau)\right|^2$)

Correlation with the code - (→ $\left|S^{-}(F_D,\tau)\right|^2$)

C. Yang, C. Hegarty, and M. Tran, "Acquisition of the GPS L5 signal using coherent combining of I5 and Q5," in Proc. of ION GNSS, 17th International Technical Meeting, Long Beach, CA, Sept. 2004, pp. 2184 – 2195.

C. J. Hegarty, "Optimal and near-optimal detector for acquisition of the GPS L5 signal," in Proc. of ION NTM, National Technical Meeting, Monterey, CA, Jan. 2006, pp. 717 – 725.

Coherent channel combining (I)



Coherent channel combining (II)

False alarm and detection probability:

$$P_{fa}(\beta)=1-\left[1-\exp\left\{-\frac{\beta}{4\sigma_n^2}\right\}\right]^2$$

$$P_d(\beta)=1-\left[1-\exp\left\{-\frac{\beta}{4\sigma_n^2}\right\}\right]\left[1-Q\left(\sqrt{\frac{\lambda}{2\sigma_n^2}},\sqrt{\frac{\beta}{2\sigma_n^2}}\right)\right]$$

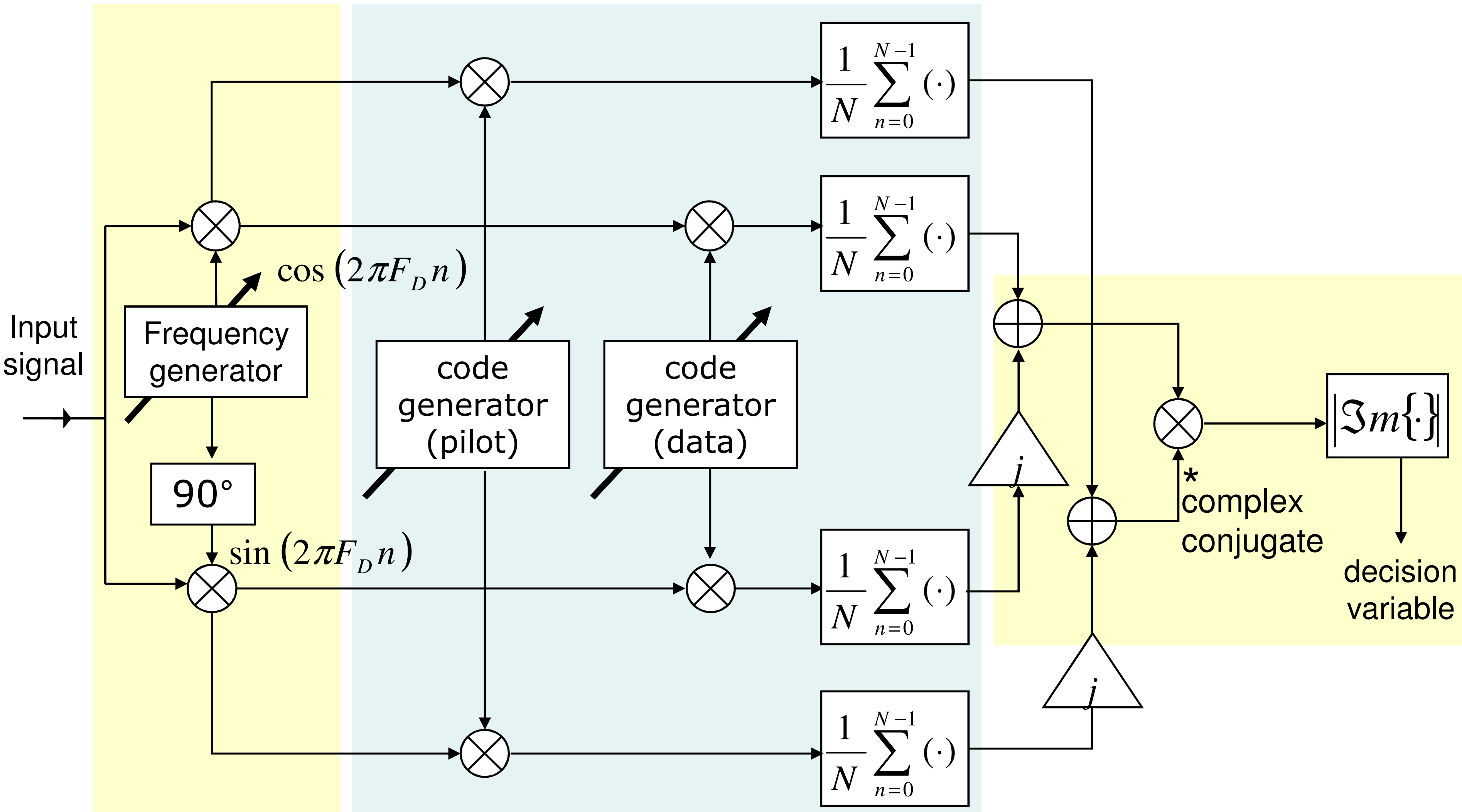
$$\approx 1-\left[1-\exp\left\{-\frac{\beta N}{N_0 f_s}\right\}\right]\left[1-Q\left(\sqrt{\frac{2CN}{N_0 f_s}},\sqrt{\frac{2\beta N}{N_0 f_s}}\right)\right]$$

Probability of false alarm for the ideal case of coherent integration

Probability of detection for the ideal case of coherent integration

PLAN

Differentially coherent channel combining (I)



J. A. A. Rodriguez, T. Pany, and B. Eissfeller, “A theoretical analysis of acquisition algorithms for indoor positioning,” in In Proc. of the 2nd ESA Workshop on Satellite Navigation User Equipment Technologies (NAVITEC), Noordwijk, The Netherlands, Dec. 2004.

Differentially coherent channel combining (II)

Decision variable:

$$S(F_D,\tau) = \left| \Im m\left\{ S^d(F_D,\tau) S^p(F_D,\tau)^* \right\} \right|$$

$$S^d(F_D,\tau) = S_I^d(F_D,\tau) + jS_Q^d(F_D,\tau)$$

$$S^p(F_D,\tau) = S_I^p(F_D,\tau) + jS_Q^p(F_D,\tau)$$

False alarm and detection probability:

$$P_{fa}(\beta) = \exp\left\{ -\frac{\beta}{\sigma_n^2} \right\}$$

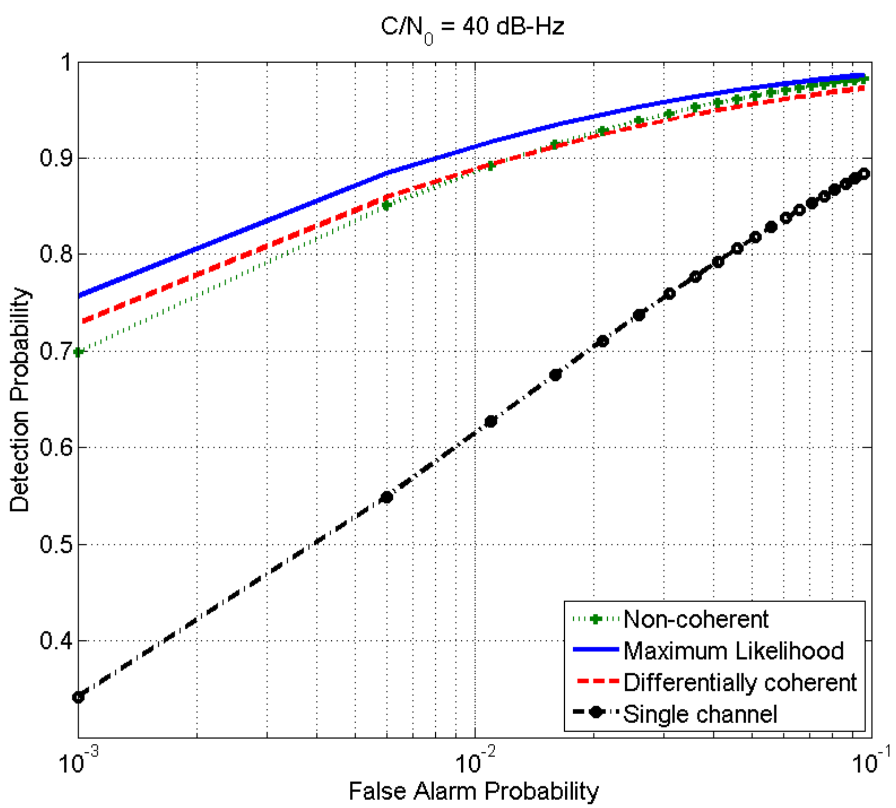
$$P_d(\beta) = \frac{1}{2}\exp\left\{ -\frac{2\beta+\lambda}{2\sigma_n^2} \right\} - \frac{1}{2}\exp\left\{ \frac{2\beta-\lambda}{2\sigma_n^2} \right\} Q\left(\frac{\sqrt{\lambda}}{\sigma_n}, \frac{2\sqrt{\beta}}{\sigma_n} \right) + Q\left(\frac{\sqrt{2\lambda}}{\sigma_n}, \frac{\sqrt{2\beta}}{\sigma_n} \right)$$

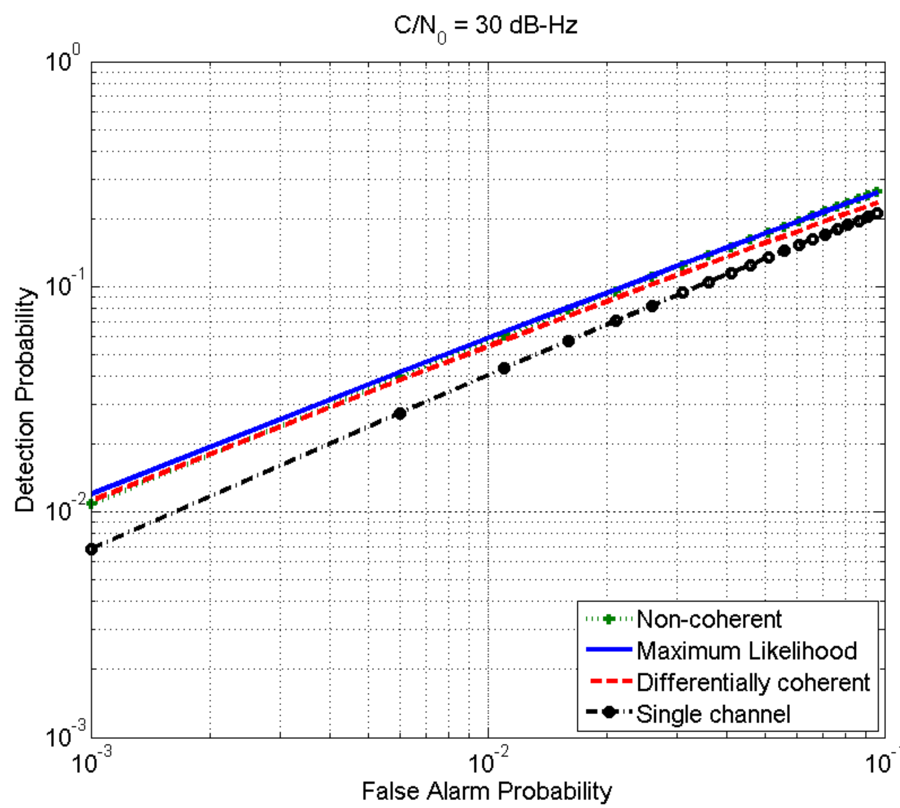
$$\lambda \approx \frac{C}{4}$$

M. K. Simon, “Probability Distributions Involving Gaussian Random Variables: A Handbook for Engineers and Scientists”, 1st ed., The International Series in Engineering and Computer Science. Springer, May 2002.

ROC results

Parameter	Value	Parameter	Value
Sampling frequency	40.92 MHz	$B_{IF}= f_s/2$	20.46 MHz
Intermediate frequency	10.23 MHz	Code Length	10230 chips
Pre-detection integration time	1 ms	Sample/chip	4





Multiple Code Period Acquisition

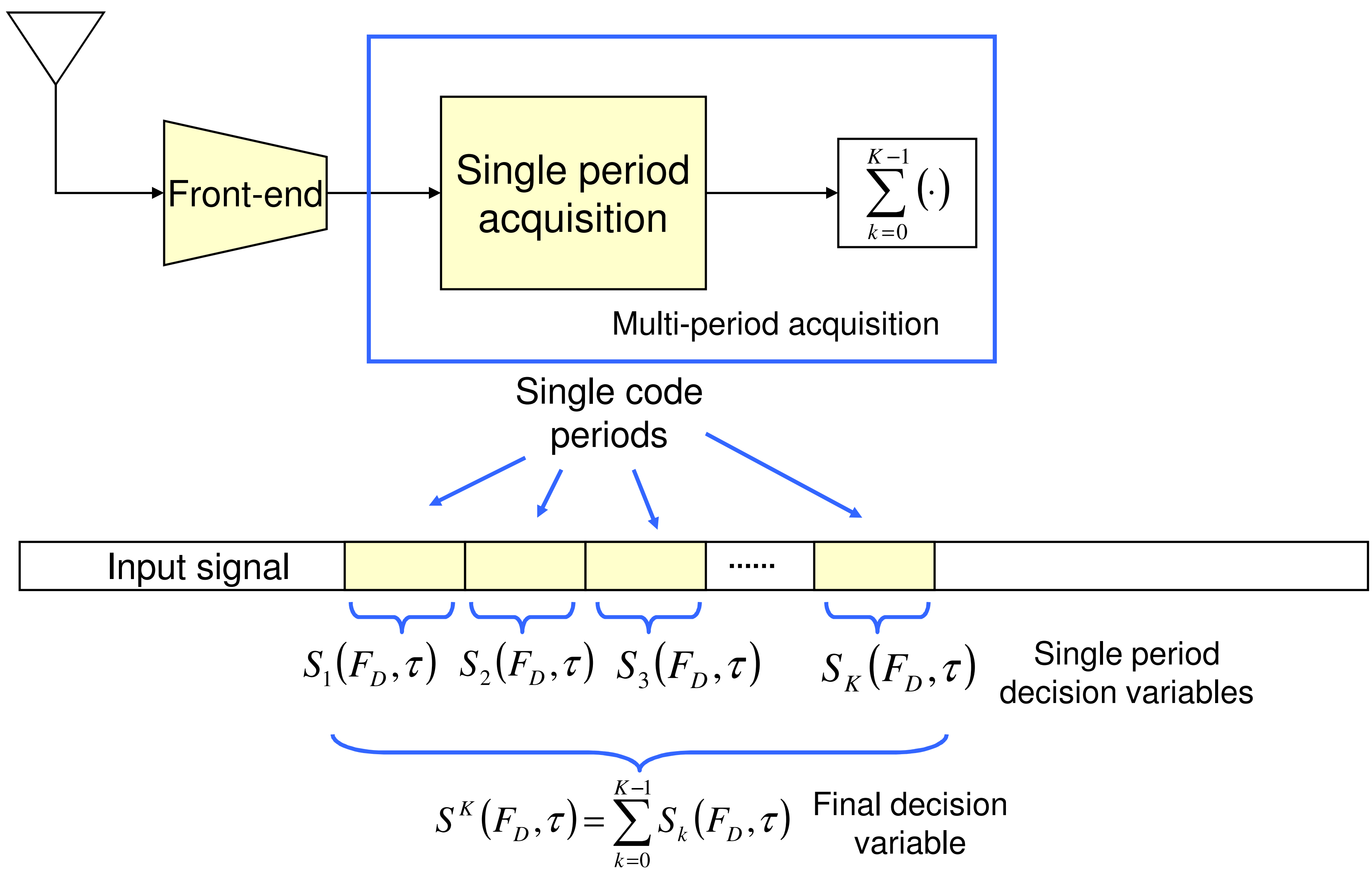
- Non-coherent combining
- Semi-coherent combining
- Differentially Coherent combining

Without sign recovery

- Exhaustive sign search
- Secondary code partial correlation

With sign recovery

Signal Acquisition without Sign recovery



Non-coherent combining

✓ **Single Channel**

$$S^K(F_D,\tau)=\sum_{i=0}^{K-1}S_k(F_D,\tau)=\sum_{i=0}^{K-1}\left[\left|S_{I,k}^x(F_D,\tau)\right|^2+\left|S_{Q,k}^x(F_D,\tau)\right|^2\right]$$

✓ **Data and Pilot**

$$S^K(F_D,\tau)=\sum_{i=0}^{K-1}\left[\left|S_{I,k}^d(F_D,\tau)\right|^2+\left|S_{Q,k}^d(F_D,\tau)\right|^2+\left|S_{I,k}^p(F_D,\tau)\right|^2+\left|S_{Q,k}^p(F_D,\tau)\right|^2\right]$$

✓ In both cases the decision variable is χ^2 square distributed with 2K and 4K degrees of freedom respectively

✓ The false alarm (central χ^2 square) and detection probabilities (non-central χ^2 square) are known from the literature

Semi-coherent combining

Decision variable:

$$S^K(F_D,\tau)=\sum_{k=0}^{K-1} S_k(F_D,\tau)=\sum_{k=0}^{K-1}\max\left\{\left|S_k^+(F_D,\tau)\right|^2,\left|S_k^-(F_D,\tau)\right|^2\right\}$$

False alarm probability:

Set of constants that can be easily determined by means of an iterative algorithm

$$P_{fa}^K(\beta)=\exp\left\{-\frac{\beta}{4\sigma^2}\right\}\sum_{i=1}^{K}\sum_{n=0}^{i-1}\left(\frac{1}{4\sigma_n^2}\right)^n\frac{1}{n!}\left[a_{K,i}-b_{K,i}2^n\exp\left\{-\frac{\beta}{4\sigma_n^2}\right\}\right]$$

The decision threshold can be determined by using a Newton-Raphson algorithm. The starting point of the algorithm can be determined by using a Gaussian approximation for the false alarm probability.

$$P_{fa}^K(\beta)\approx\frac{1}{2}\operatorname{erfc}\left(\frac{\beta-6K\sigma_n^2}{\sqrt{2\cdot 10K\sigma_n^4}}\right)\quad \text{for } K>>1$$

C. Yang, C. Hegarty, and M. Tran, "Acquisition of the GPS L5 signal using coherent combining of I5 and Q5," in Proc. of ION GNSS, 17th International Technical Meeting, Long Beach, CA, Sept. 2004, pp. 2184 – 2195.

Differentially Coherent combining

Decision variable:

$$S^K(F_D,\tau)=\sum_{k=0}^{K-1}\left|\Im m\left\{S_k^d(F_D,\tau)S_k^p(F_D,\tau)^*\right\}\right|$$

False alarm probability:

$$P_{fa}^K(\beta)=\exp\left\{-\frac{\beta}{\sigma_n^2}\right\}\sum_{i=0}^{K-1}\left(\frac{\beta}{\sigma_n^2}\right)^i$$

Some remarks

✓ All these techniques remove the bit dependence by means of a non-linear operation (squaring, absolute value …);

✓ The size of the Doppler bin doesn't have to be reduced since the coherent integration time is constant and is equal to 1 ms;

✓ The non-coherent (dual channel), the semi-coherent and the differentially coherent combining require similar computational loads.

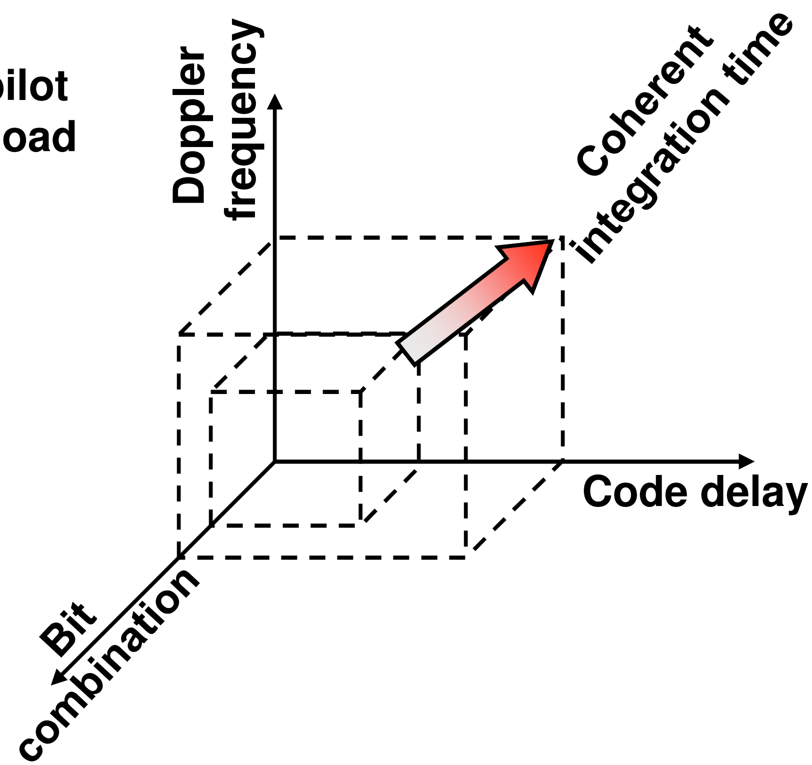
Acquisition with sign recovery

The coherent integration time can be increased by estimating the sequence of bits that modulates the data and pilot channels.

All methods that try to estimate the data and pilot bit sequences require a heavy computational load since, as the integration time increases,

- **there are more bit combinations to be tested**

- **the size of the Doppler bin has to be reduced accordingly**



Two strategies:

✓ **exhaustive sign combinations search;**

✓ **partial secondary code correlations**

Exhaustive sign search

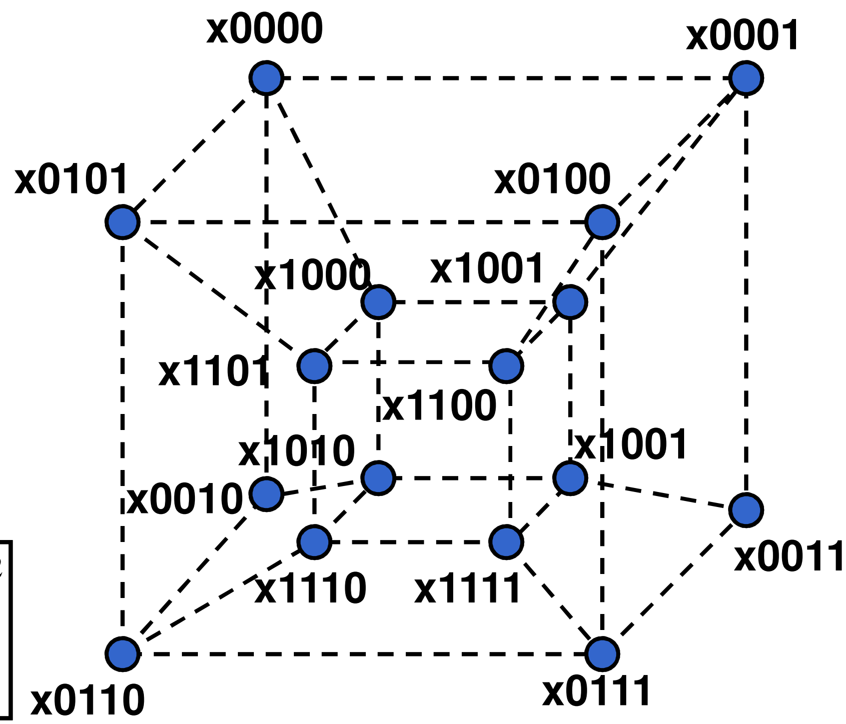
Decision variable:

$$S^K(F_D,\tau)=\max_{D_K}\left|\sum_{k=0}^{K-1}\left[d_{d,k}S_k^d(F_D,\tau)+jd_{p,k}S_k^p(F_D,\tau)\right]\right|^2$$

$$D_K=\left\{d_{d,k},d_{p,k}\right\}_{k=0}^{K-1}$$

Set of the possible sign combinations of data and pilot components

By ignoring the secondary code → 2^{2K-1} **possible bit combinations**



✓ The number of bit combinations grows exponentially with K.

✓ The method estimates the bit sequence for both pilot and data channels:

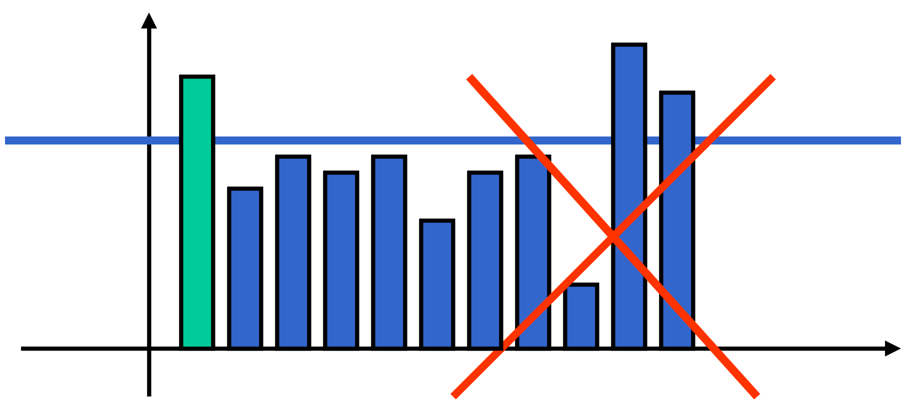
$$\hat{D}_K=\arg\max_{D_K}\left|\sum_{k=0}^{K-1}\left[d_{d,k}S_k^d(F_D,\tau)+jd_{p,k}S_k^p(F_D,\tau)\right]\right|^2$$

Secondary code partial correlation

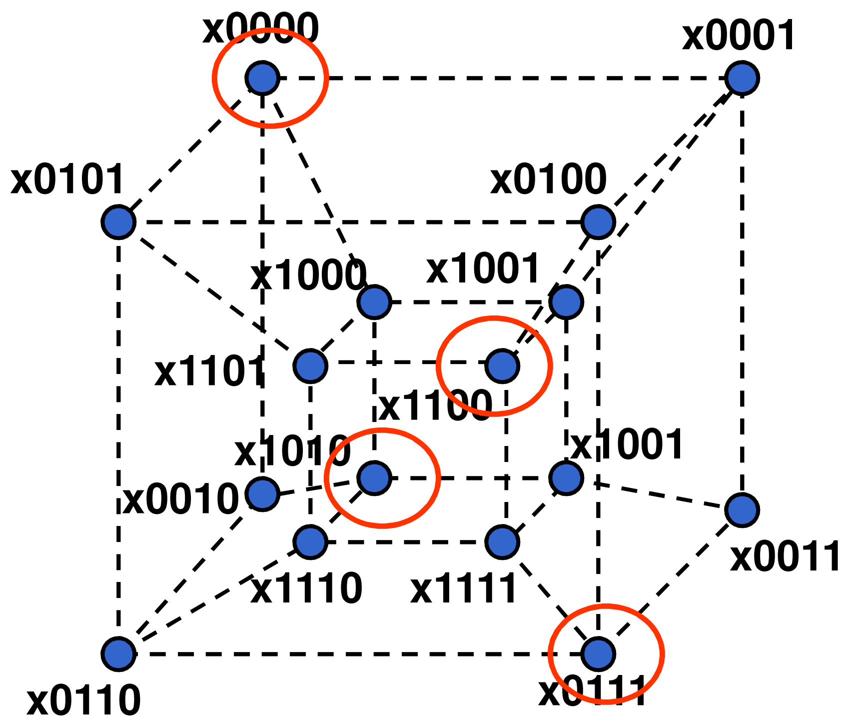
```
 +                                +                         -
…0 1 0 0 1 | 1 0 0 0 0 1 0 0 0 0 1 0 1 1 1 0 1 0 0 1 | 0 1 1 1 1 …
  1 0 0 1 1  0 0 0 0 1 0 0 0 0 1 0 1 1 1 0 1 0 0 1  0 1 1 1 1
  0 0 1 1 0  0 0 0 1 0 0 0 0 1 0 1 1 1 0 1 0 0 1  0 1 1 1 1
  0 1 1 0 0  0 0 1 0 0 0 0 1 0 1 1 1 0 1 0 0 1  0 1 1 1 1
  ……………………………………………………………
```

Only specific bit combinations are allowed by secondary codes

✓reduced computational load;
✓improved system performance, since less candidates implies less opportunity to have a false alarm



It excludes candidates that can lead to a false alarm



Sign combinations

Number of possible bit combinations (single channel)

$$N_d + K - 1$$

Length of the data secondary code

Number of possible bit combinations (dual channel combining)

$$2H(N_d + K - 1) \qquad H = \frac{N_p}{N_d}$$

Length of the pilot secondary code

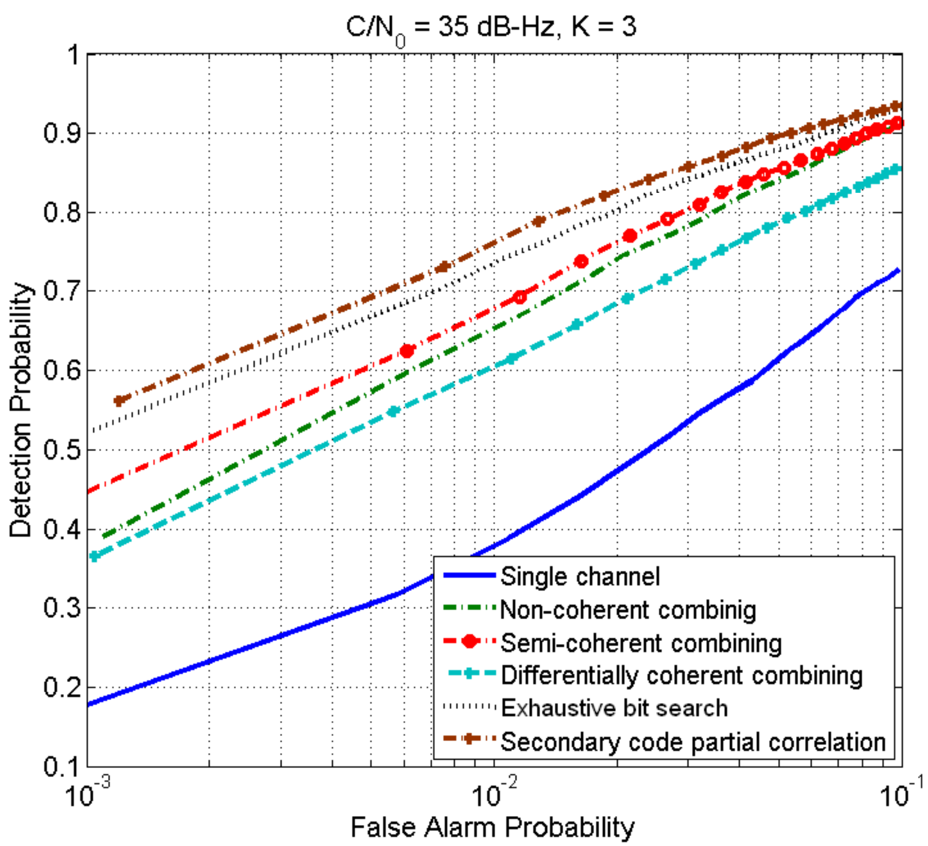
For increasing the coherent integration time (till to 20 ms) it is more convenient to use the **data channel alone, exploiting the properties of its secondary code.**

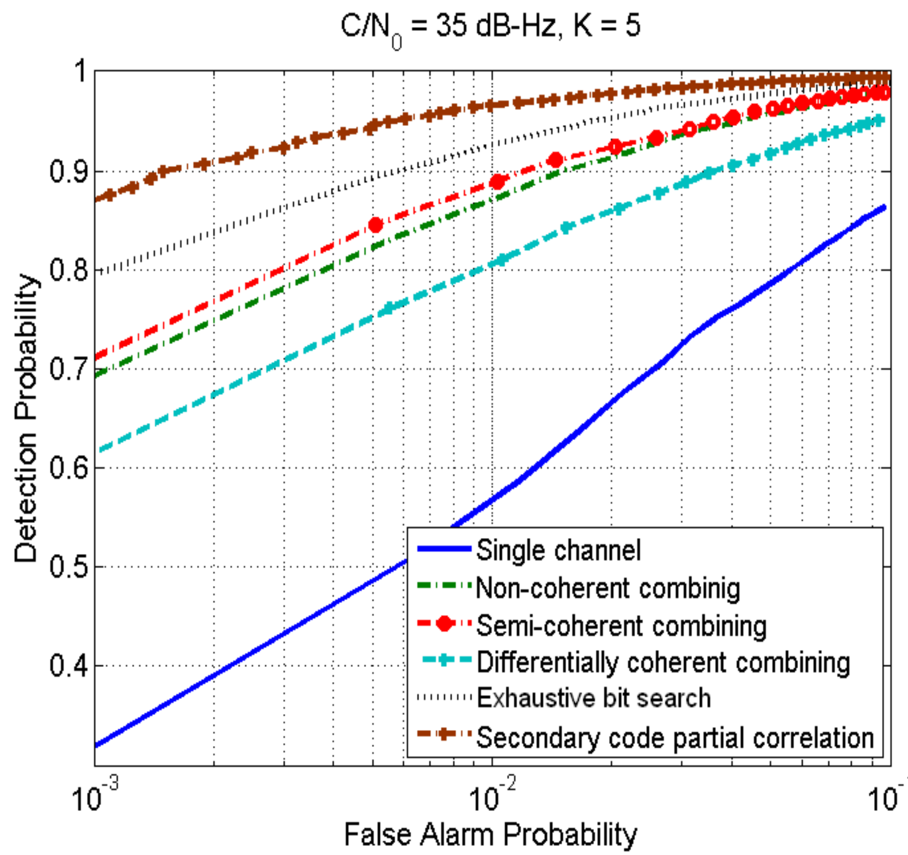
K	Data + Pilot channels	Data channel*	Exhaustive search
2	210	23	8
3	220	25	32
4	230	27	128
5	240	29	512
6	250	31	2048
7	260	33	8192
8	270	35	32768
9	280	37	131072

* in order to have a fair comparison with the other two cases the integration time for the data channel alone has been doubled

ROC results

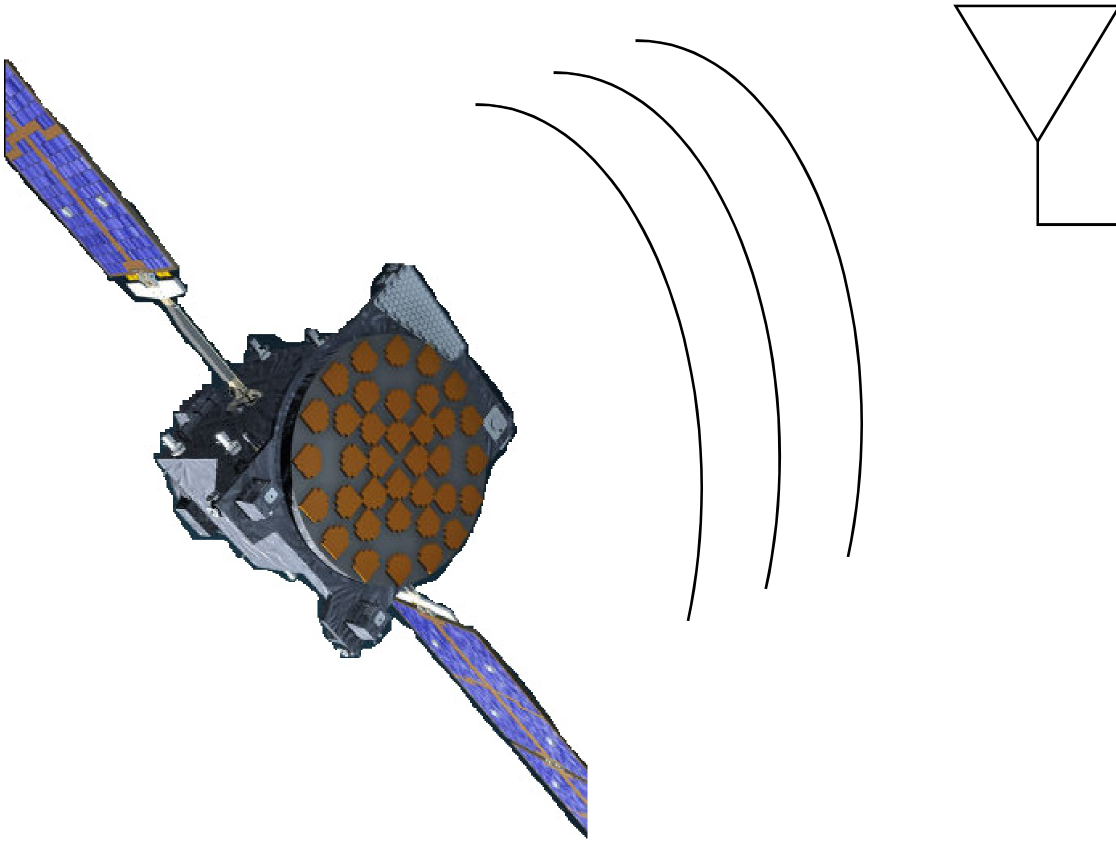
Parameter	Value	Parameter	Value
Sampling frequency	40.92 MHz	$B_{IF}= f_s/2$	20.46 MHz
Intermediate frequency	10.23 MHz	Code Length	10230 chips
Pre-detection integration time	1 ms	Sample/chip	4

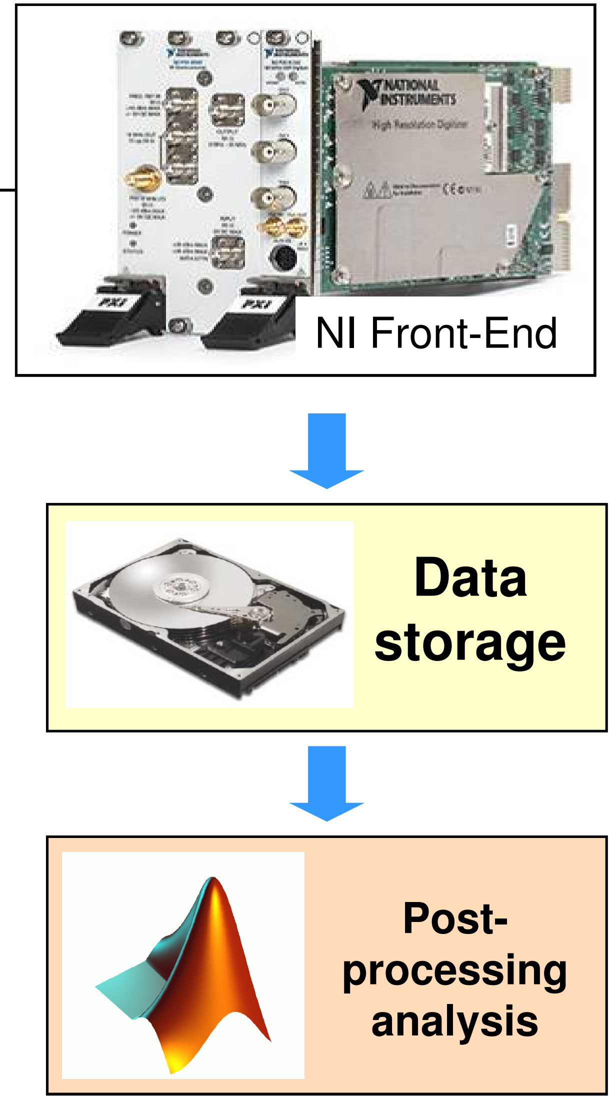




Real data analysis

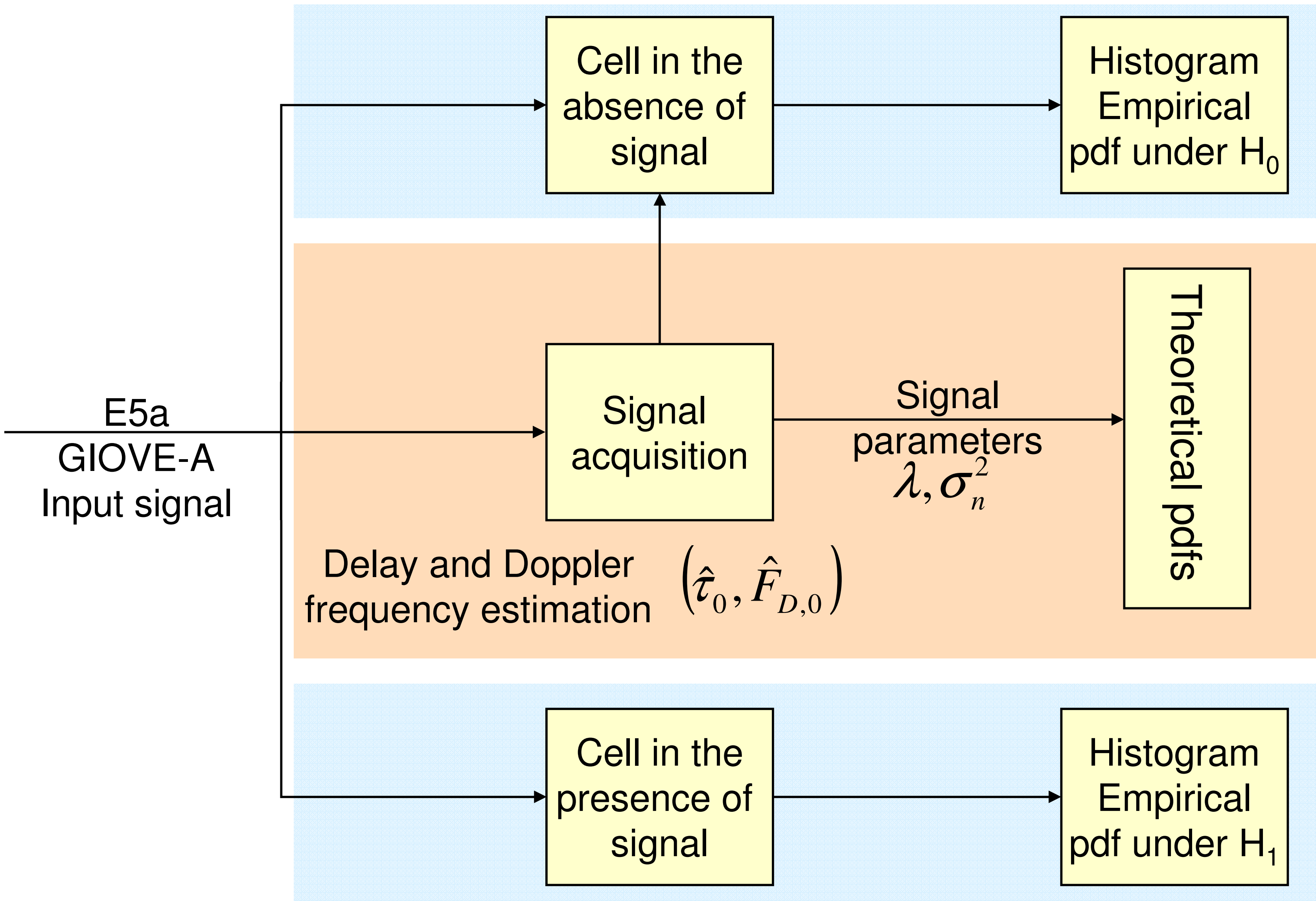
Experimental Setup



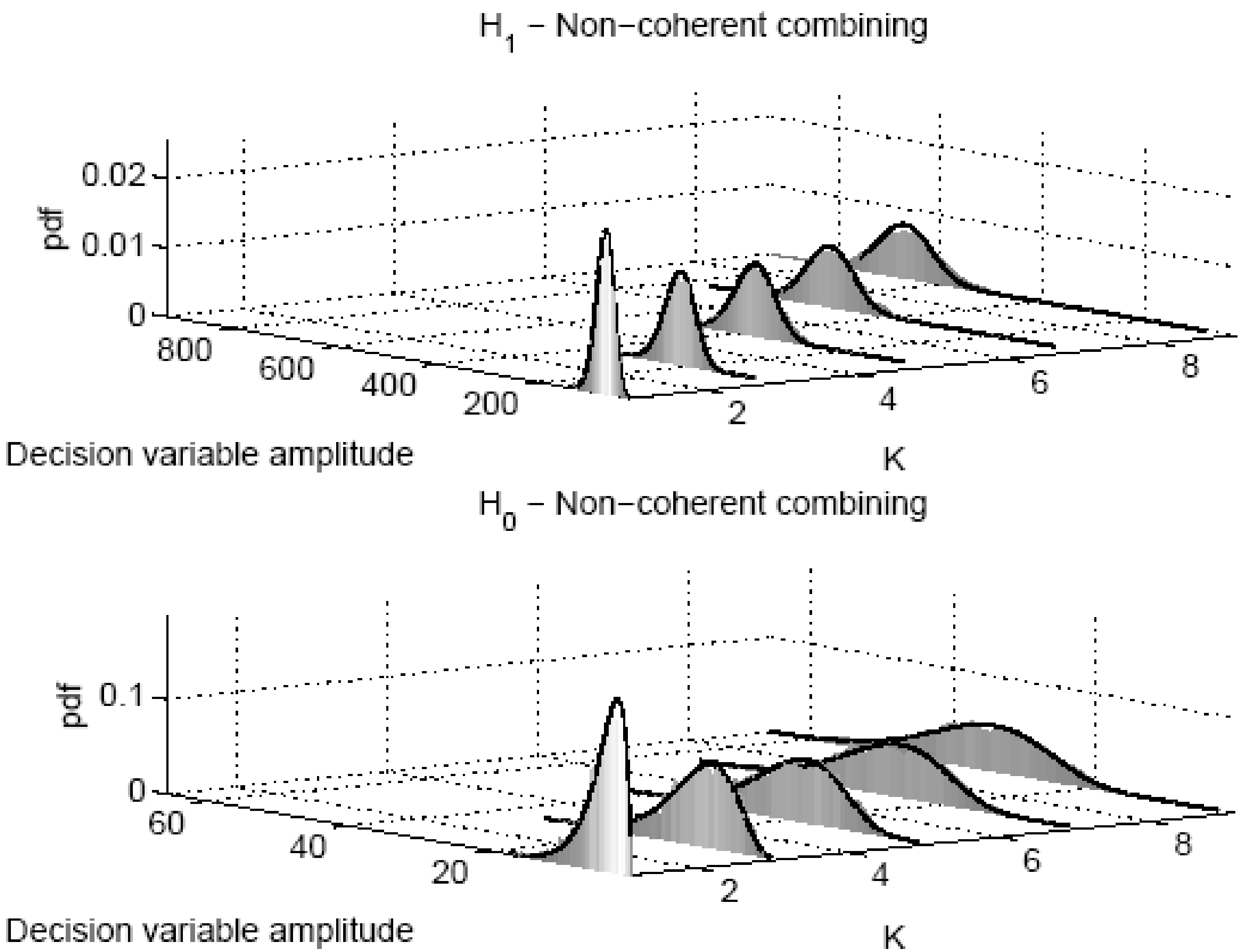


Live data from GIOVE-A, the first Galileo experimental satellite, have been collected by using the NI PXI-5661 signal analyzer and used to test the acquisition algorithms proposed.

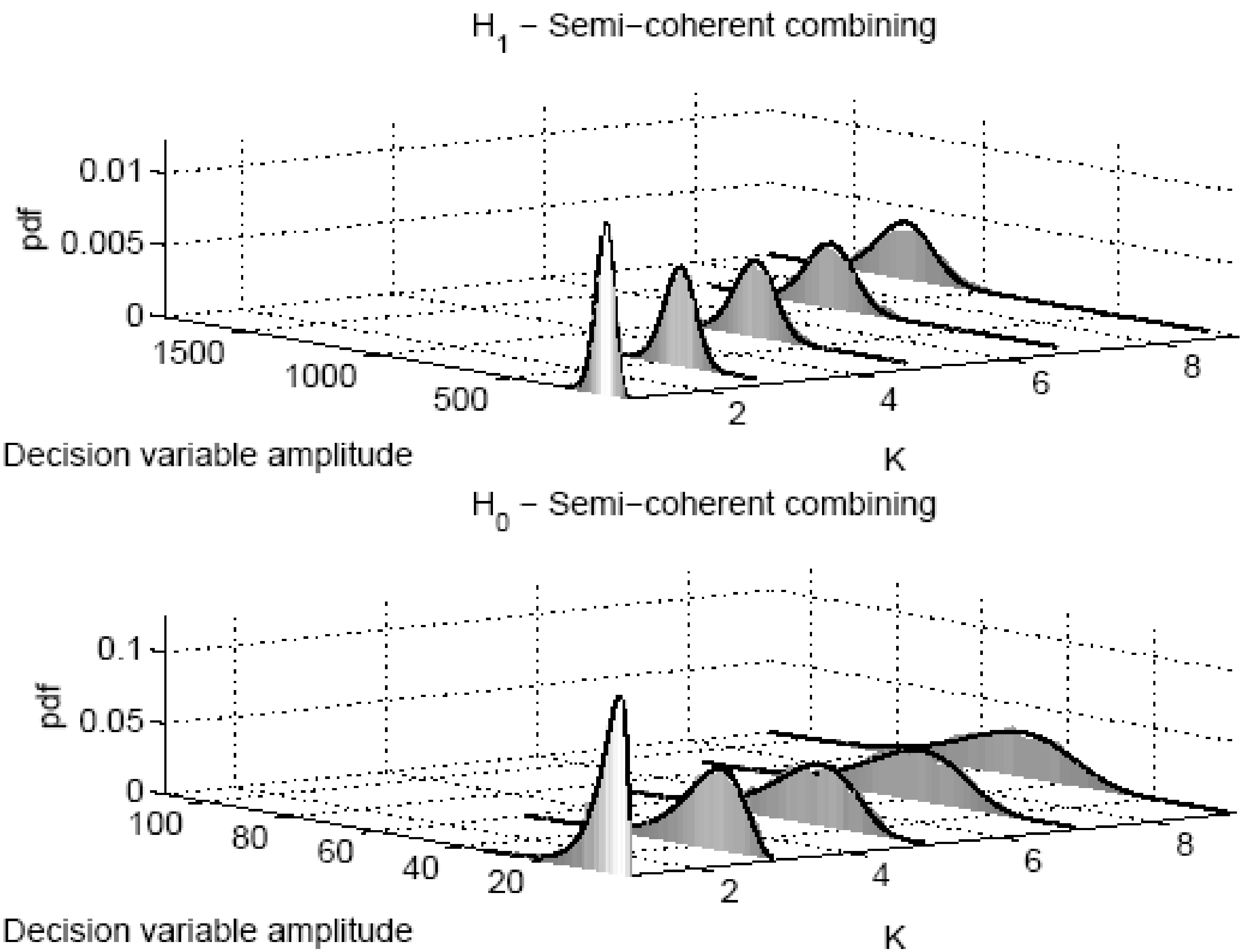
Analysis principle



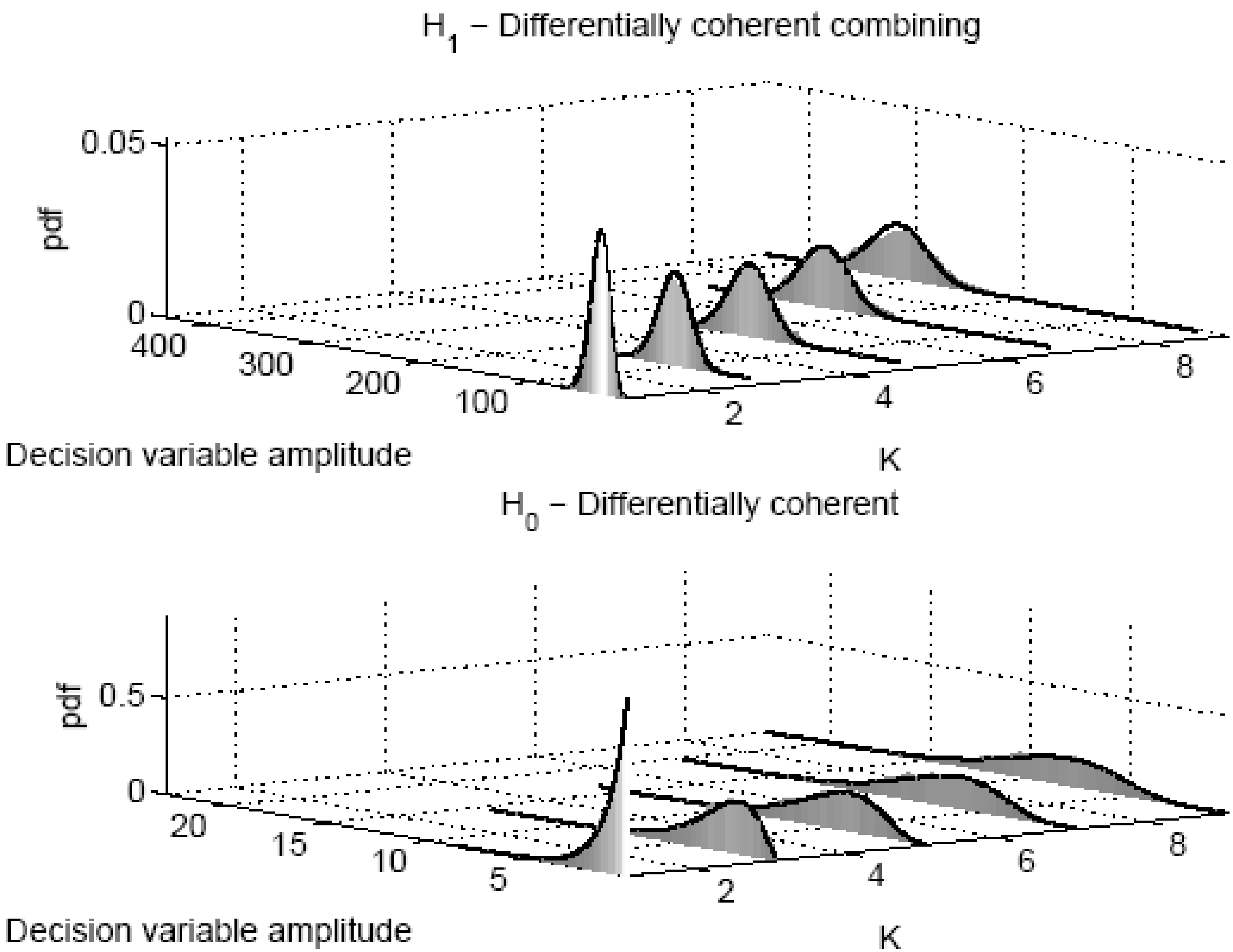
Non-coherent combining



Semi-coherent combining



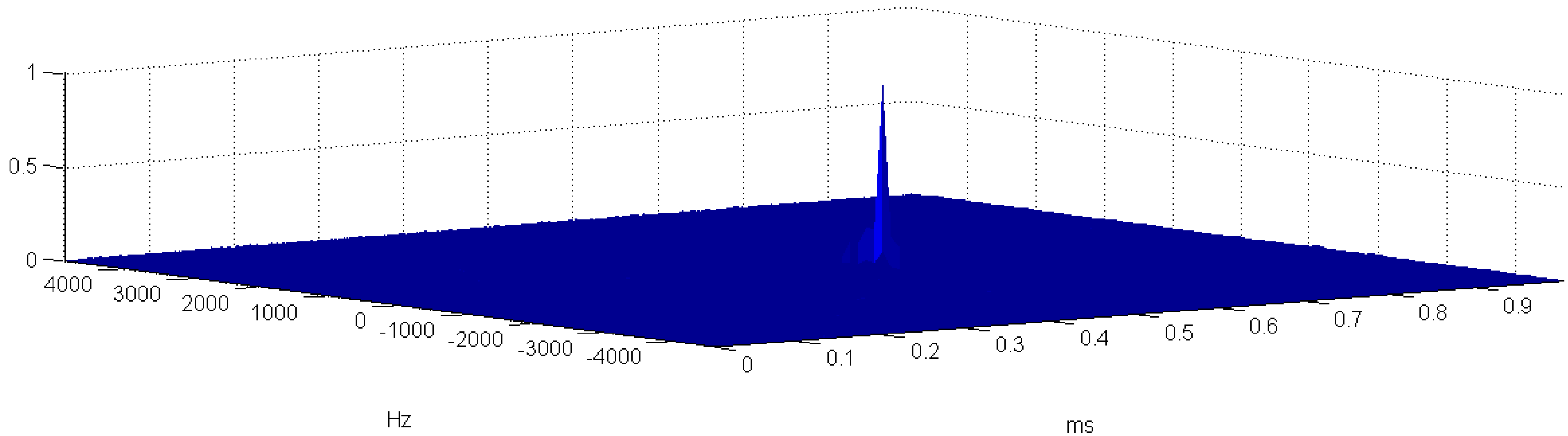
Differentially coherent combining



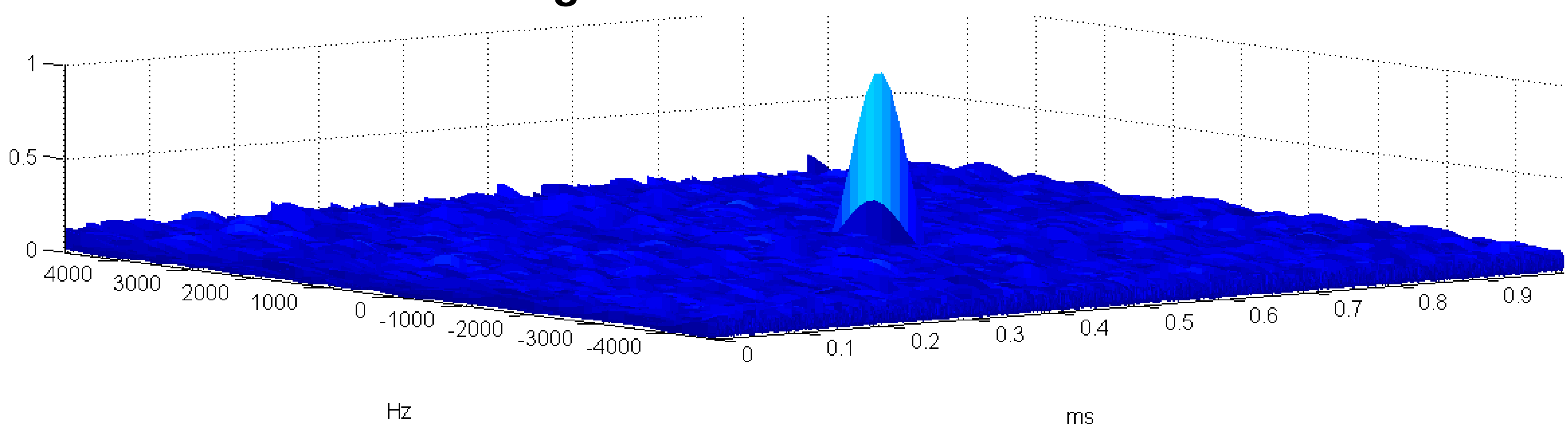
Coherent vs. non-coherent

K = 5

Coherent combining



Non-coherent combining



Conclusions

• When the acquisition on a single code period is considered the coherent channel combining with sign recovery results the more effective acquisition strategy. For low C/N_0 the sign estimation is no more reliable and coherent and non-coherent channel combining tends to have the same performance.

• When considering acquisition on multiple code periods two classes of algorithms can be identified: with and without sign recovery.

• The pure non-coherent, the semi-coherent and the differentially coherent combining belong to the first class, and require a reduced computational load with respect to the other strategies since the sign combinations have not to be searched for and the Doppler bin size has not to be reduced. Among these strategies the semi-coherent integration gives better performance for high C/N_0. For low C/N_0 semi-coherent and non-coherent integrations lead to similar performances.

• Among the second class, the secondary code partial correlation outperforms all the other techniques requiring a lower computational load with respect to the exhaustive search of all the possible bit combinations.

PLAN