GPS Self-Interference and Mitigation Methods

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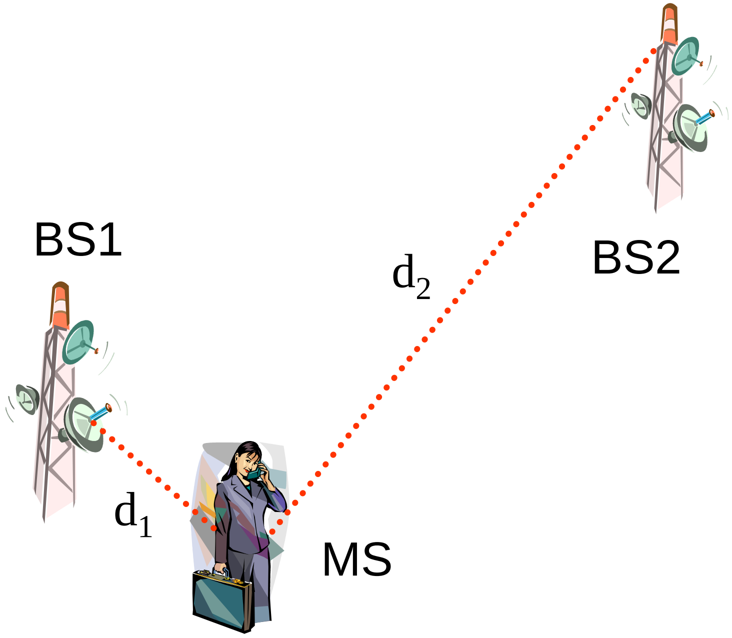
Self Interference: Near-Far Problem

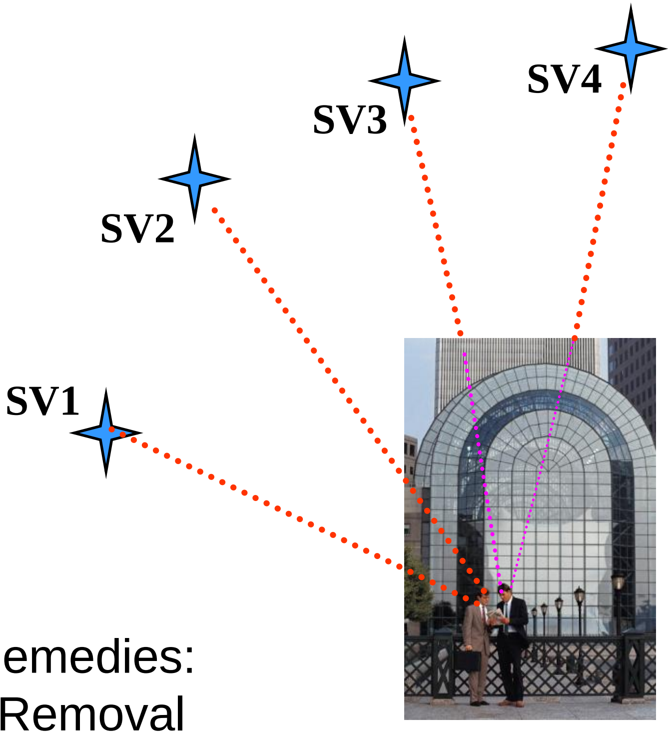
Communication systems:
$d_1 << d_2$, $P_{BS1} >> P_{BS2}$



Remedies:
- Adaptive control of power
- Data rates & modulation

GPS:
All satellite distances differ < 20%

Urban environment:
Attenuation > multi-access gain



Remedies:
- Removal
-Extended Integration
-……

Signal Model

Problem Statement

Self-Interference Assessment

Subspace Projection Method

Simulation Results

$$\underset{\text{Input}}{\mathbf{y}} = \underset{\substack{\text{Strong}\\ \text{signals}\\ (m)}}{\mathbf{S}\boldsymbol{\alpha}_s} + \underset{\substack{\text{Weak}\\ \text{signals}\\ (k)}}{\mathbf{W}\boldsymbol{\alpha}_w} + \underset{\text{Noise}}{\mathbf{v}}$$

$$\mathbf{y} = [y(1) \quad y(2) \quad \cdots \quad y(N)]^T$$

$$\mathbf{v} = [v(1) \quad v(2) \quad \cdots \quad v(N)]^T$$

$$\boldsymbol{\alpha}_s = [\alpha_{s1} \quad \alpha_{s2} \quad \cdots \quad \alpha_{sm}]^T \qquad \boldsymbol{\alpha}_w = [\alpha_{w1} \quad \alpha_{w2} \quad \cdots \quad \alpha_{wk}]^T$$

$$\mathbf{S} = [\mathbf{s}_1 \quad \mathbf{s}_2 \quad \cdots \quad \mathbf{s}_m] \in C^{N\times m} \qquad \mathbf{W} = [\mathbf{w}_1 \quad \mathbf{w}_2 \quad \cdots \quad \mathbf{w}_k] \in C^{N\times k}$$

$$\mathbf{s}_i = [s_i(1) \quad s_i(2) \quad \cdots \quad s_i(N)]^T \qquad \mathbf{w}_j = [w_j(1) \quad w_j(2) \quad \cdots \quad w_j(N)]^T$$

$$s(k) = c_{si}(k) d_{si}(k) e^{j(2\pi f_{dsi} k\tau + \phi_{0si})} \qquad w_j(k) = c_{wj}(k) d_{wj}(k) e^{j(2\pi f_{dwj} k T_s + \phi_{0wj})}$$

$c_{si}(k)$, $c_{wj}(k)$: PRN codes

$d_{si}(k)$, $d_{wj}(k)$: Navigation data bits

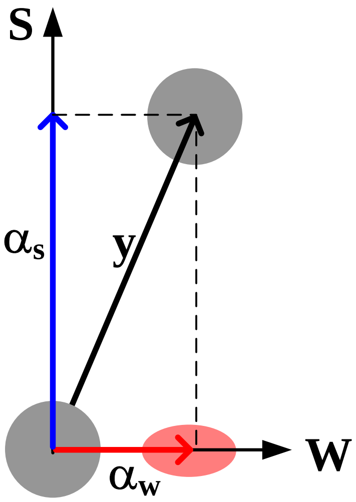
$f_{dsi}(k)$, $f_{dwj}(k)$: Doppler frequencies

$\phi_{0si}(k)$, $\phi_{0wi}(k)$: Initial phases

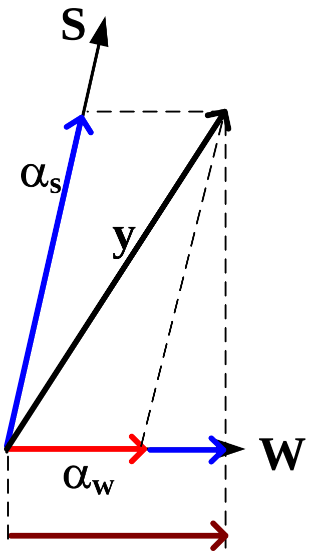
Masking Effects: Cross-Correlations

Orthogonal Signals

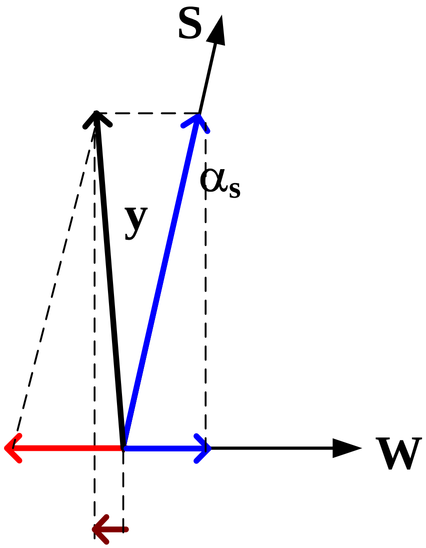
Non-Orthogonal Signals



No-cross correlation



Constructive cross-correlation



Destructive cross-correlation

Receiver Acquisition Process

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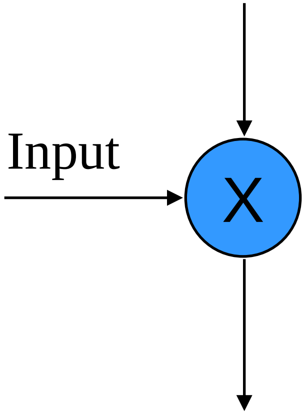
Simulation Results

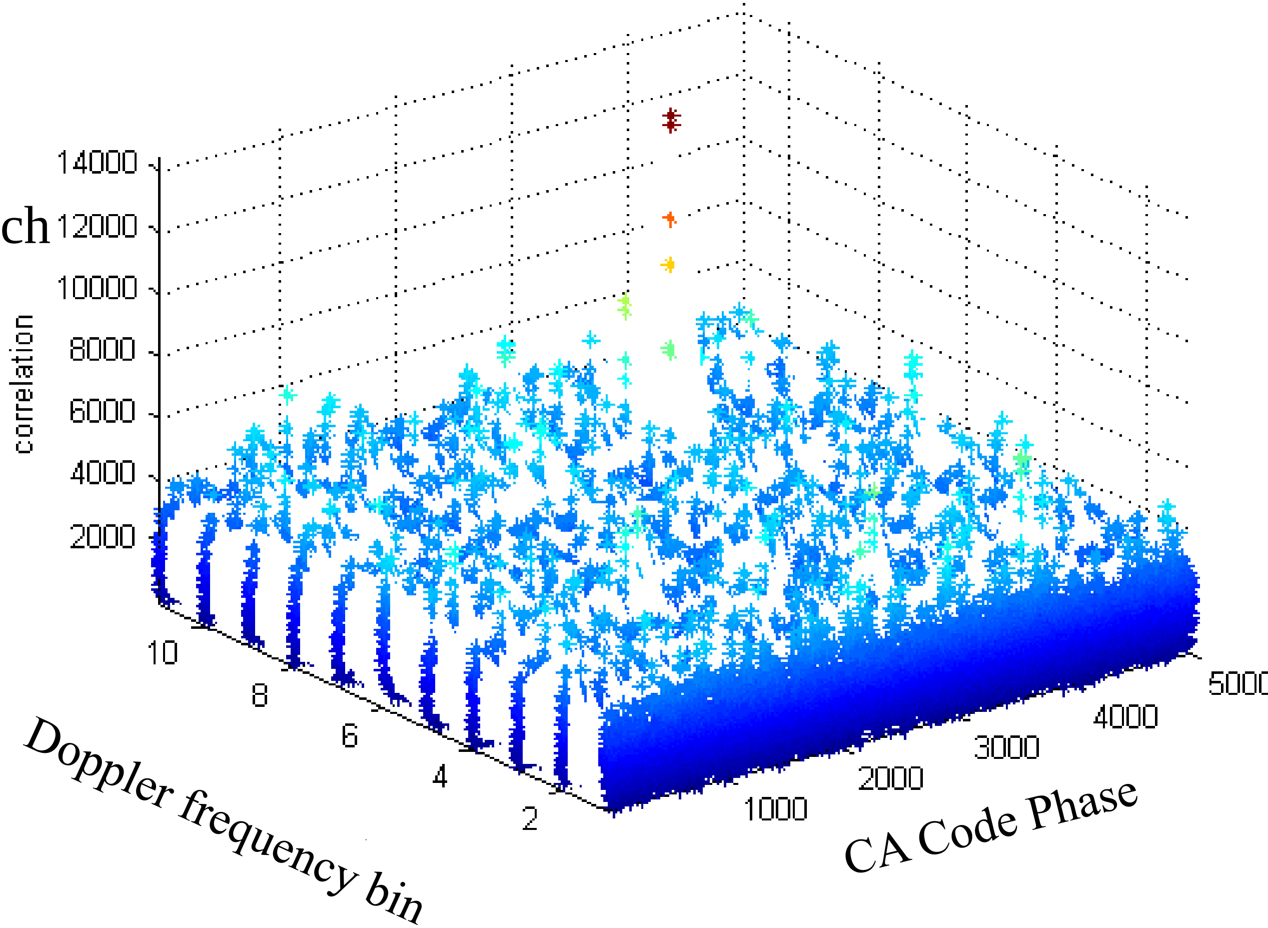


Reference $r_w = c_w(t-\hat{t}_w)e^{-j\hat{\omega}_{dw}t}$

Input

$$R_w(\Delta t_w, \Delta\omega_{dw}) = \int_T x(t) r_w(t) dt$$

Find a match



Correlation Computation Analysis

$$x(t) \otimes r_w(t) = (\alpha_s s + \alpha_w w + v) \otimes r_w(t)$$

Strong-weak signal cross-correlation:

$$\alpha_s s \otimes r_w = \{c_s(t - t_s)e^{j\omega_{ds}t}\} \otimes \{c_w(t - \widehat{t}_w)e^{-j\widehat{\omega}_{dw}t}\}$$

Weak signal autocorrelation:

$$\alpha_w w \otimes r_w = \{c_w(t - t_w)e^{j\omega_{dw}t}\} \otimes \{c_w(t - \widehat{t}_w)e^{-j\widehat{\omega}_{dw}t}\}$$

1. $\alpha_w w \otimes r_w(t) \approx \max \left\| \alpha_s s \otimes r_w \right\|$
2. $\alpha_s s \otimes r_w(t)$ effectively raises noise floor

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$$\alpha_w w \otimes r_w(t) \approx \max \left\| \alpha_s s \otimes r_w \right\|$$

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Simulation Results

Normalized correlation value	Probability
-65/1023	12.5%
-1/1023	75%
63/1023	12.5

$$\alpha_w \approx \frac{65}{1023} \alpha_s$$

$$P_s - P_w = 20 \log_{10} \frac{\alpha_s}{\alpha_w} \approx 23.9dB$$

Additional factors:

1. Sampling frequency inaccuracy increases higher cross correlation peak by 1.5 dB at $f_s = 5$ MHz
2. Doppler offset further increases cross correlation peak by 1.6 dB

$$P_s\text{-}P_w = 23.9 - 1.5 - 1.6 = 20.8 \text{ dB}$$

Normal Strong-Weak Signal Power Margin

- Satellite transmission antenna gain pattern
- Atmospheric path attenuation differences
- SV age differences
- Receiver antenna gain pattern

Normal $C/N_0 = \boxed{34 \sim 52}$ dB-Hz

↓

Normal range: $18 \text{ dB} < \Delta = 20.8 \text{ dB}$

Successful acquisition possible for all satellite in direct view

$\alpha_s s \otimes r_w(t)$ Raises Noise Floor

Cross-correlation power:

$$N_c = E\{(\alpha_s s \otimes r_w)^2\}$$

$$= P_s E\{(c_s \otimes c_w)^2\}$$

$$C = E\{(c_s \otimes c_w)^2\} = 0.0007$$

$$\boxed{N_c = 0.0007\ P_s}$$

Cross-Correlation Impact on Effective Input SNR

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Simulation Results

G = acquisition process gain
N= input noise power

Input weak signal SNR: $SNR_w = 10\log \frac{P_w}{N}$

Post-acq. without strong signal: $SNR_w^{'} = 10\log \frac{P_w}{N/G}$

Post-acq. include strong signal: $SNR_w^{'} = 10\log \frac{P_w}{N_c + N/G}$

$$\boxed{SNR_w^{'} = SNR_w + G_{dB} - 10\log(CG10^{\frac{SNR_s}{10}} + 1)}$$

Cross-Correlation Reduces Effective SNR_w

Problem Statement

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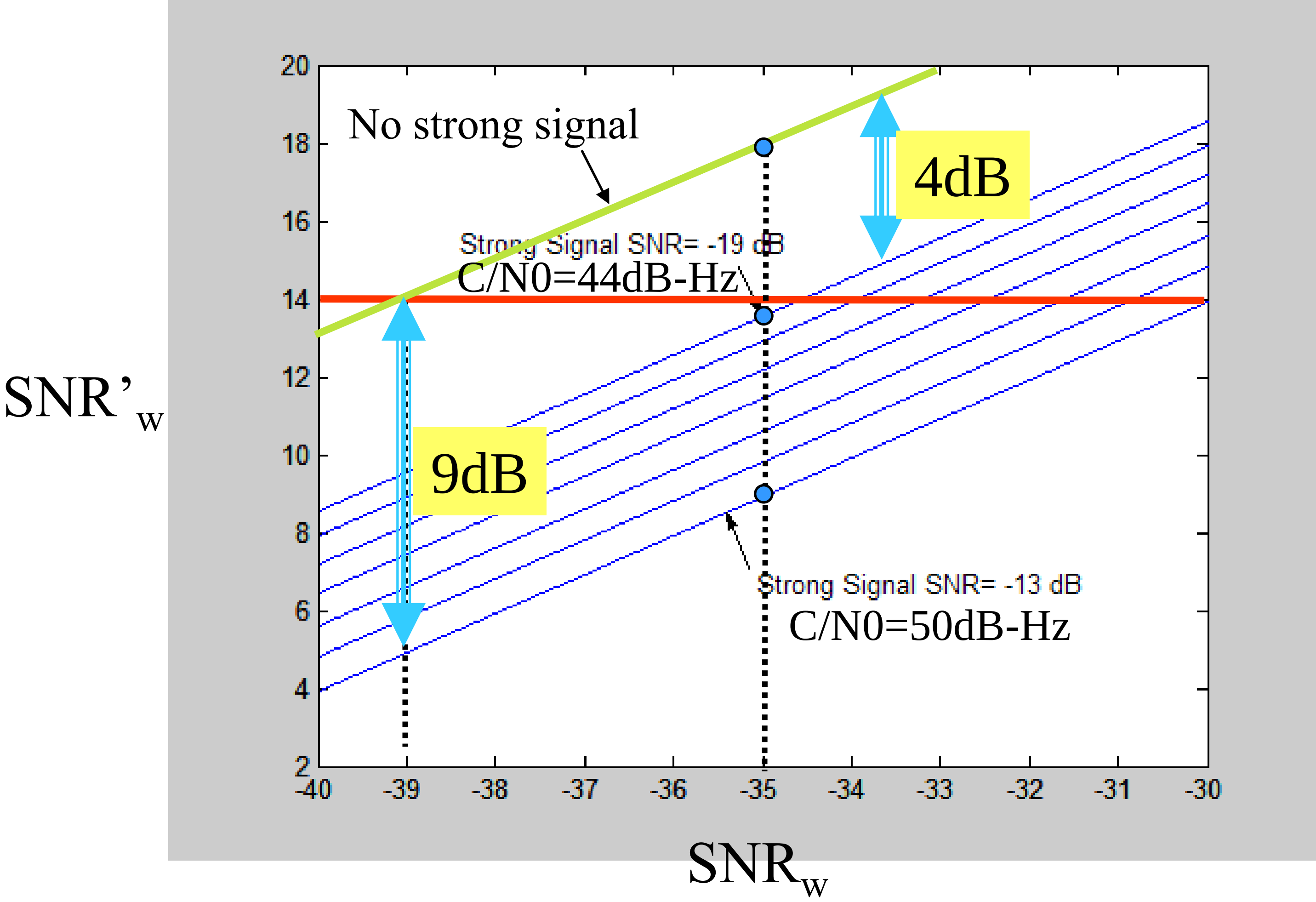
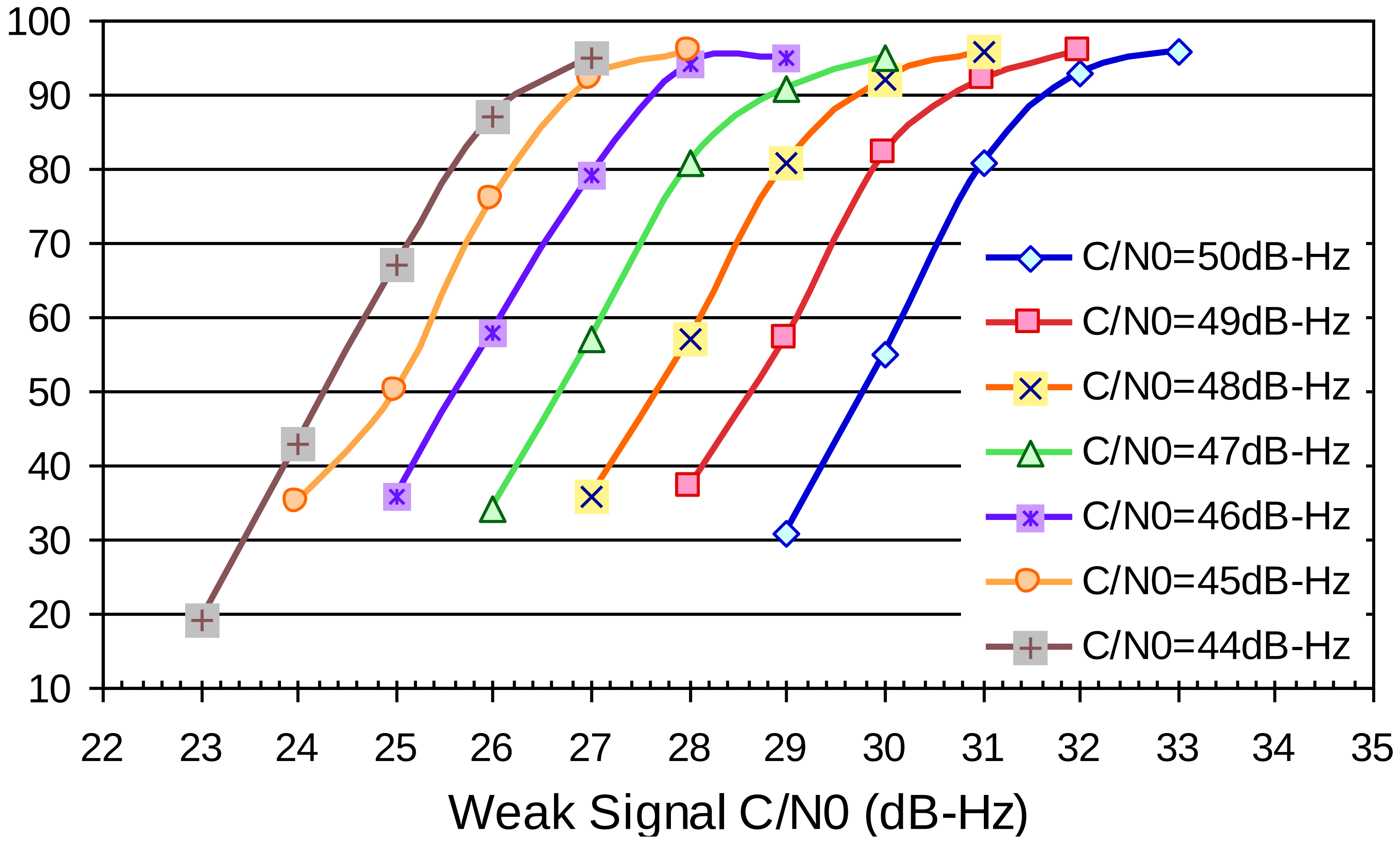
Simulation Results

Assume G_{dB} =53 dB



Weak Signal Acquisition Success Rate

1 Strong Signal Present
Based on 500 Simulation Runs



Subspace Projection Method

$$\boxed{P_s = S(S^{\mathrm{T}}S)^{-1}S^{\mathrm{T}}}$$

$$\mathbf{y} = \mathbf{S}\boldsymbol{\alpha}_s + \mathbf{W}\boldsymbol{\alpha}_w + \mathbf{v}$$

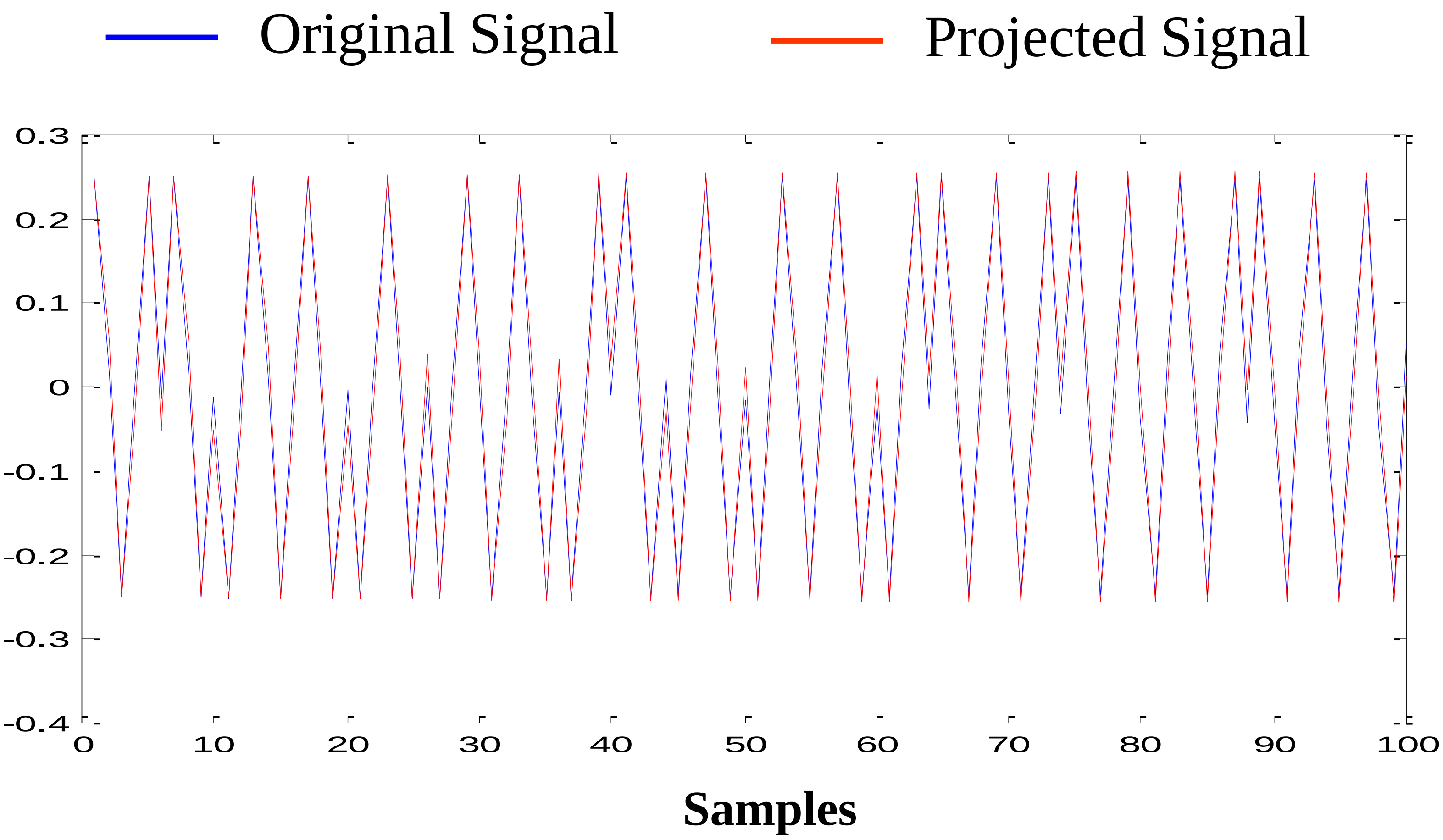
$$\begin{aligned} P_s y &= S(S^{\mathrm{T}}S)^{-1}S^{\mathrm{T}}(\mathrm{W}\alpha_w + \mathrm{S}\alpha_s + \mathrm{v}) \\ &= S(S^{\mathrm{T}}S)^{-1}(S^{\mathrm{T}}\mathrm{W}\alpha_w + S^{\mathrm{T}}\mathrm{S}\alpha_s) + P_s\mathrm{v} \end{aligned}$$

$$S^{\mathrm{T}}.\mathbf{W}\boldsymbol{\alpha}_{\mathbf{w}} << S^{\mathrm{T}}.\mathbf{S}\boldsymbol{\alpha}_{\mathbf{ws}}$$

$$P_s y \cong S\alpha_s + P_s\mathrm{v}$$

$$\mathrm{y} - P_s\mathrm{y} \cong a_w\mathrm{W} + P_s\mathrm{v}$$

Projection Operation Accuracy



Doppler Frequency on Code Shift

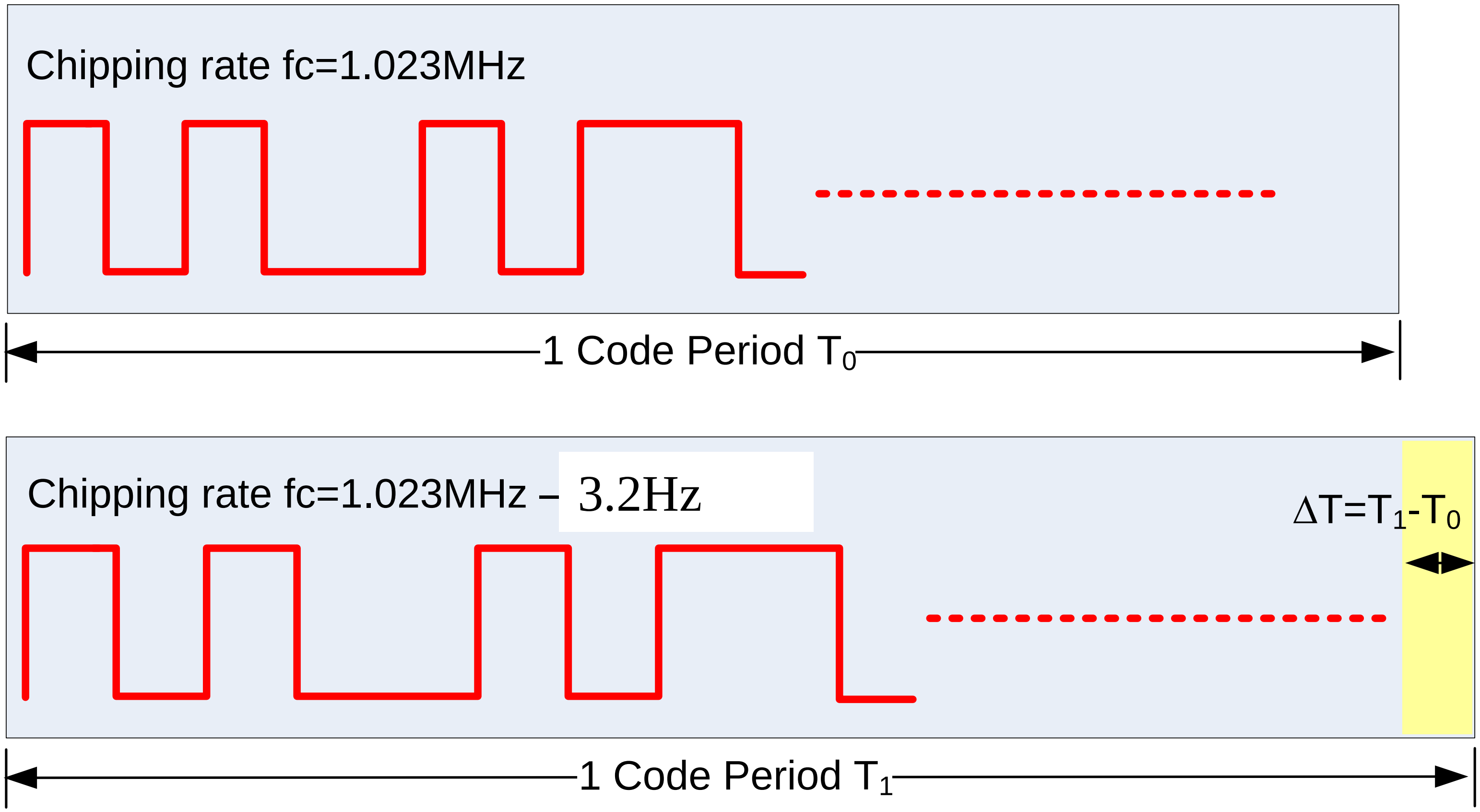
Problem Statement

Self-Interference Assessment

Subspace Projection Method

Simulation Results

L1 CA code frequency: f_{CA}=1.023 MHz
Maximum possible Doppler shift: f_{dCA}=3.2 Hz



Chipping rate fc=1.023MHz

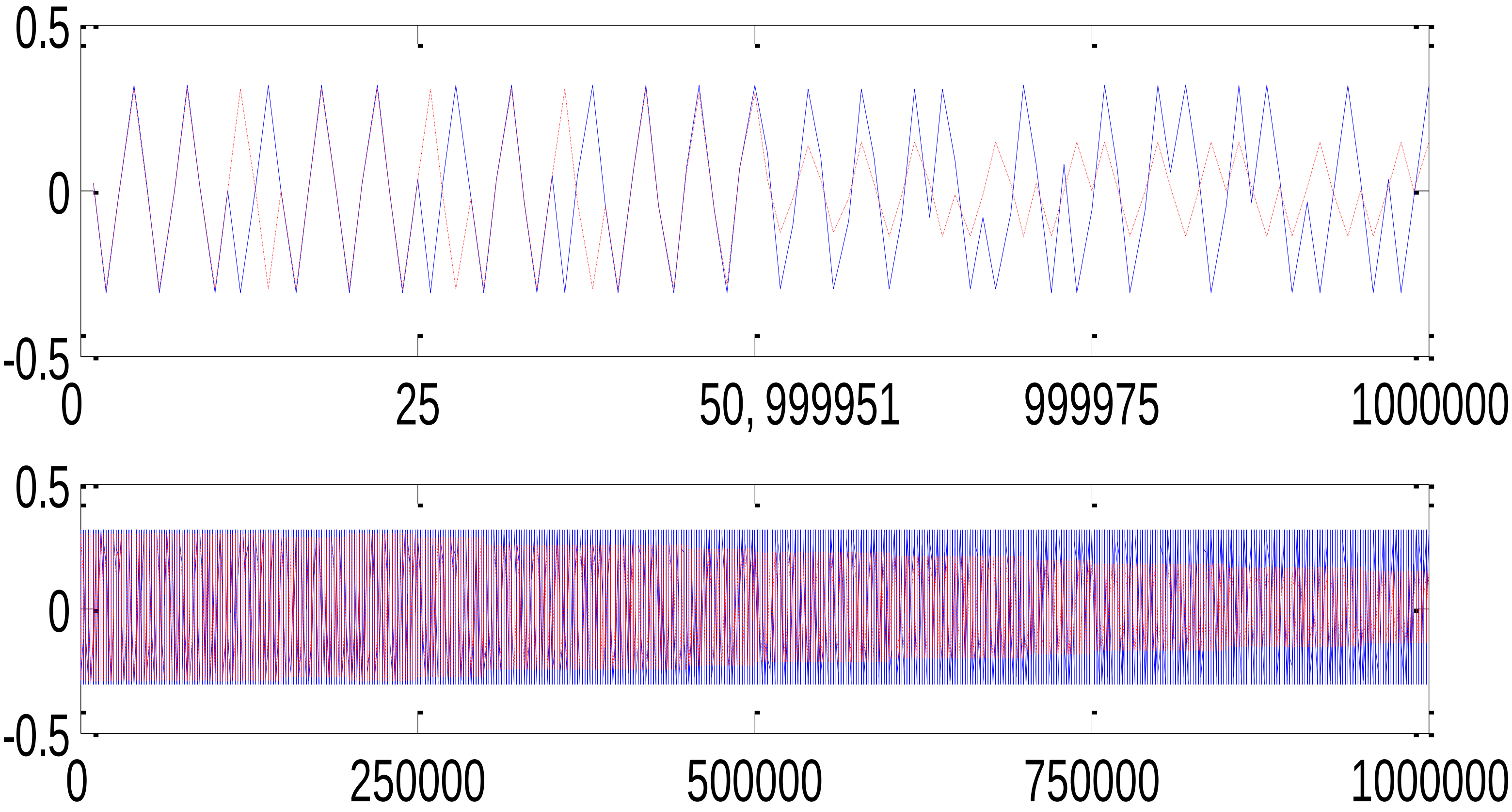
1 Code Period T_0

Chipping rate fc=1.023MHz – 3.2Hz

$\Delta T = T_1 - T_0$

1 Code Period T_1

Doppler Shift Impact on Projection



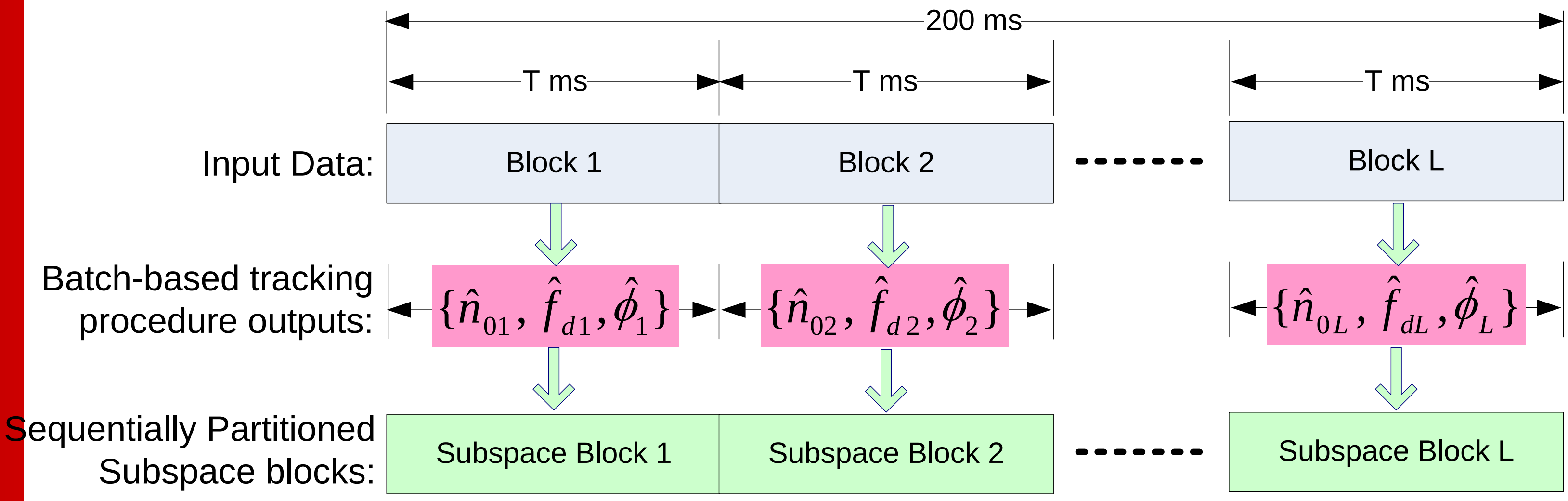
Partitioned Subspace Projection

Problem Statement

Self-Interference Assessment

Subspace Projection Method

Simulation Results



200 ms

T ms T ms T ms

Input Data: Block 1 | Block 2 | ------- | Block L

Batch-based tracking procedure outputs: $\{\hat{n}_{01}, \hat{f}_{d1}, \hat{\phi}_1\}$ | $\{\hat{n}_{02}, \hat{f}_{d2}, \hat{\phi}_2\}$ | $\{\hat{n}_{0L}, \hat{f}_{dL}, \hat{\phi}_L\}$

Sequentially Partitioned Subspace blocks: Subspace Block 1 | Subspace Block 2 | ------- | Subspace Block L

Block Processing to Reduce Doppler Shift Impact

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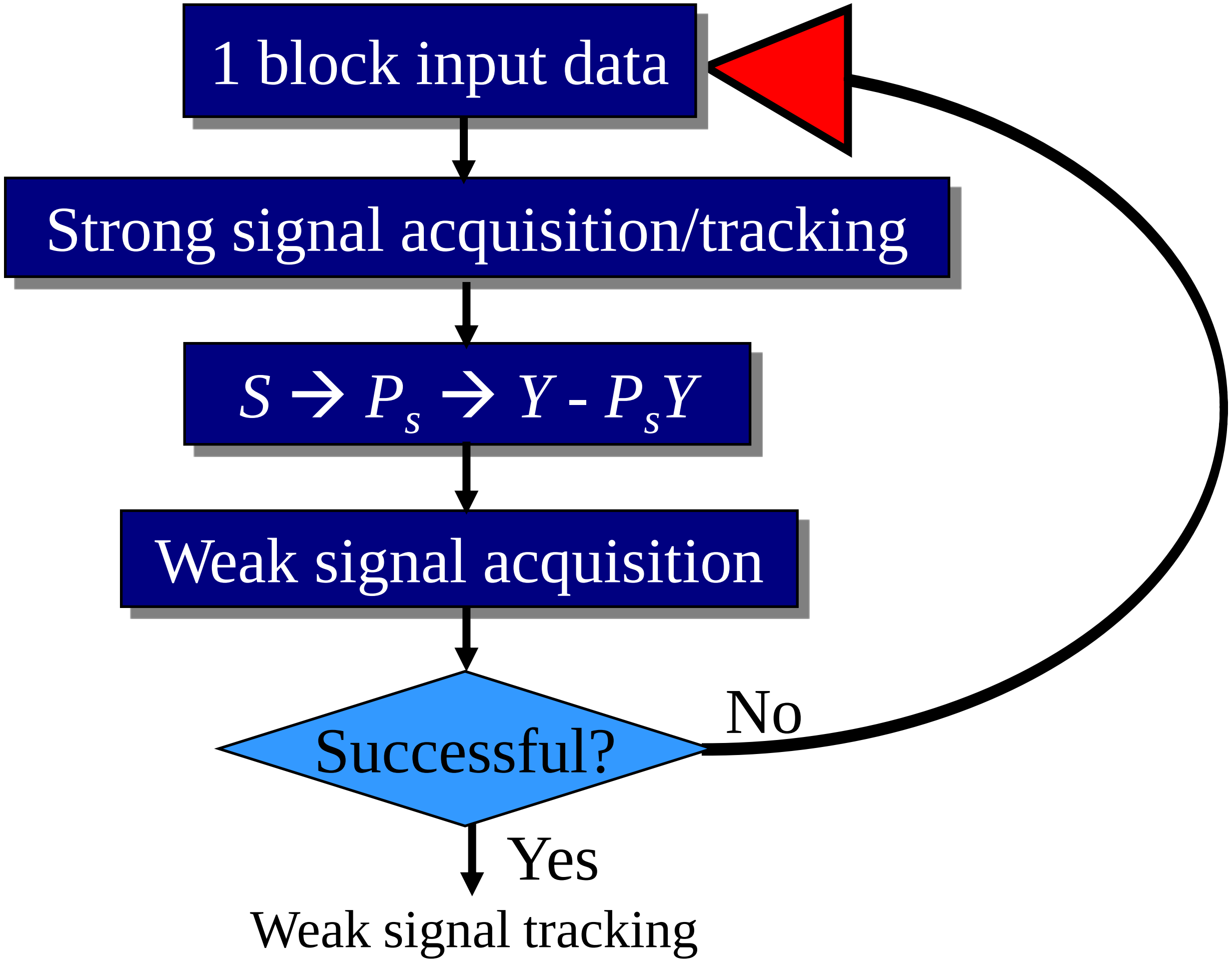

1 block input data

Strong signal acquisition/tracking

$S \rightarrow P_s \rightarrow Y - P_s Y$

Weak signal acquisition

Successful?

No

Yes

Weak signal tracking

Block Processing Improvement on Doppler Impact

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Self-Interference Assessment

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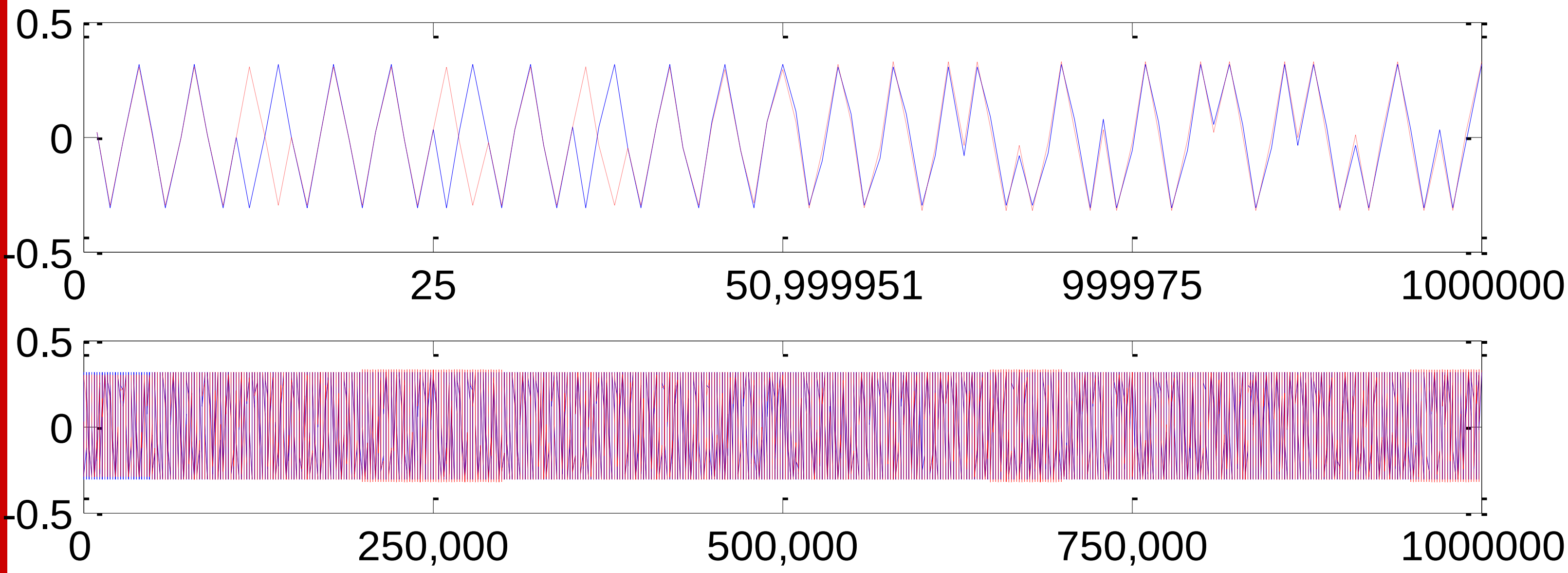

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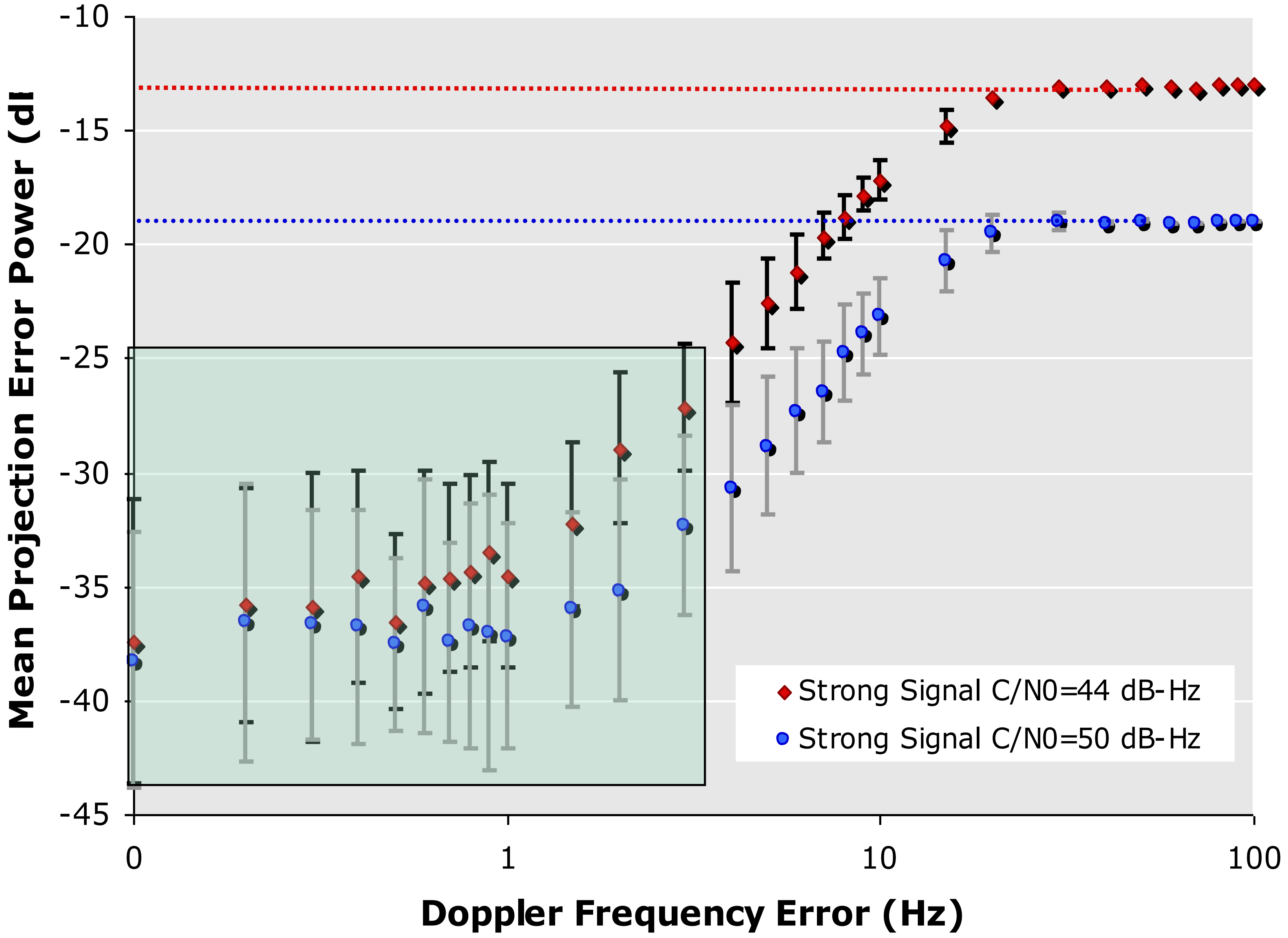
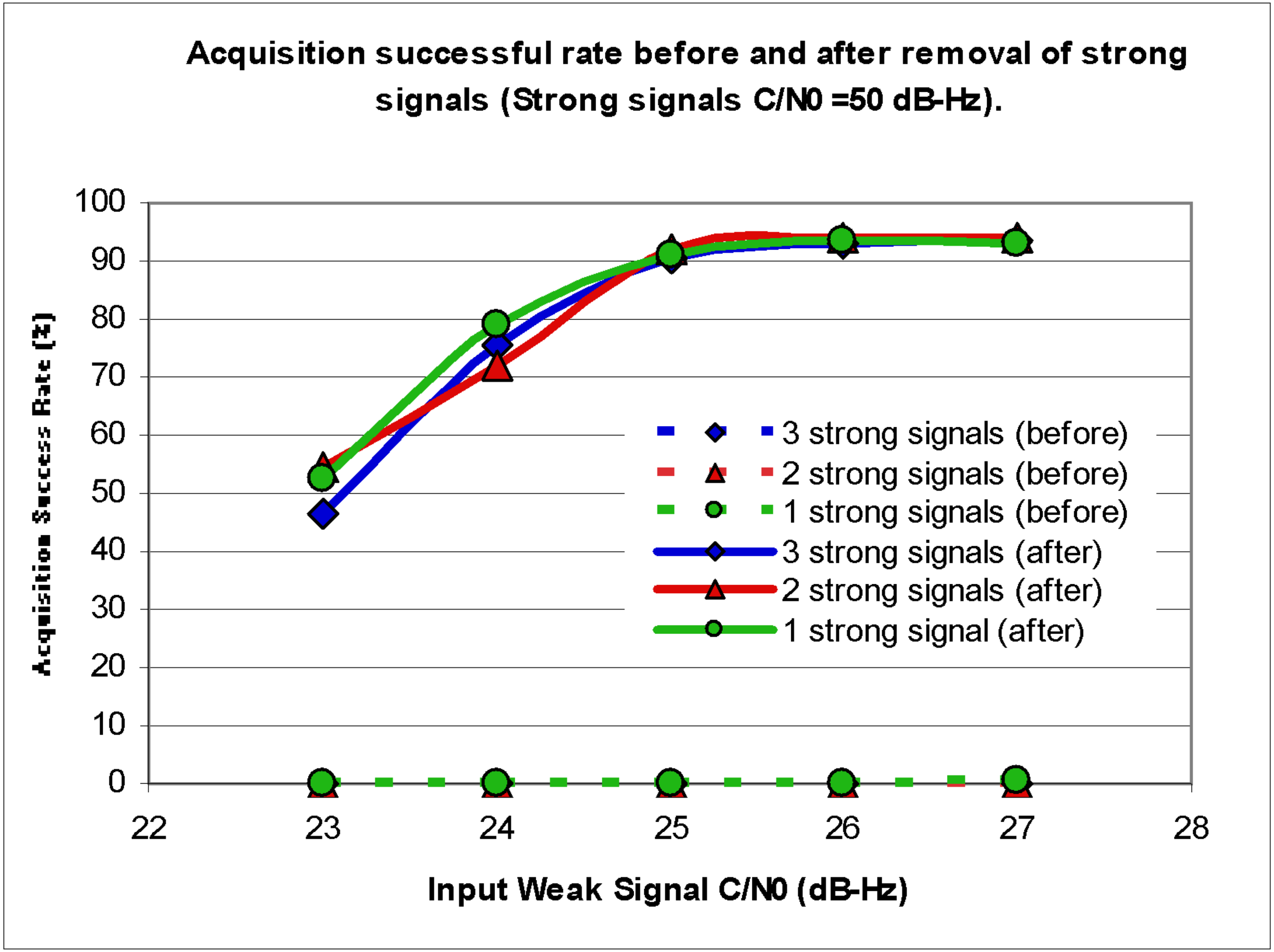
Block Processing Projection Error Simulation

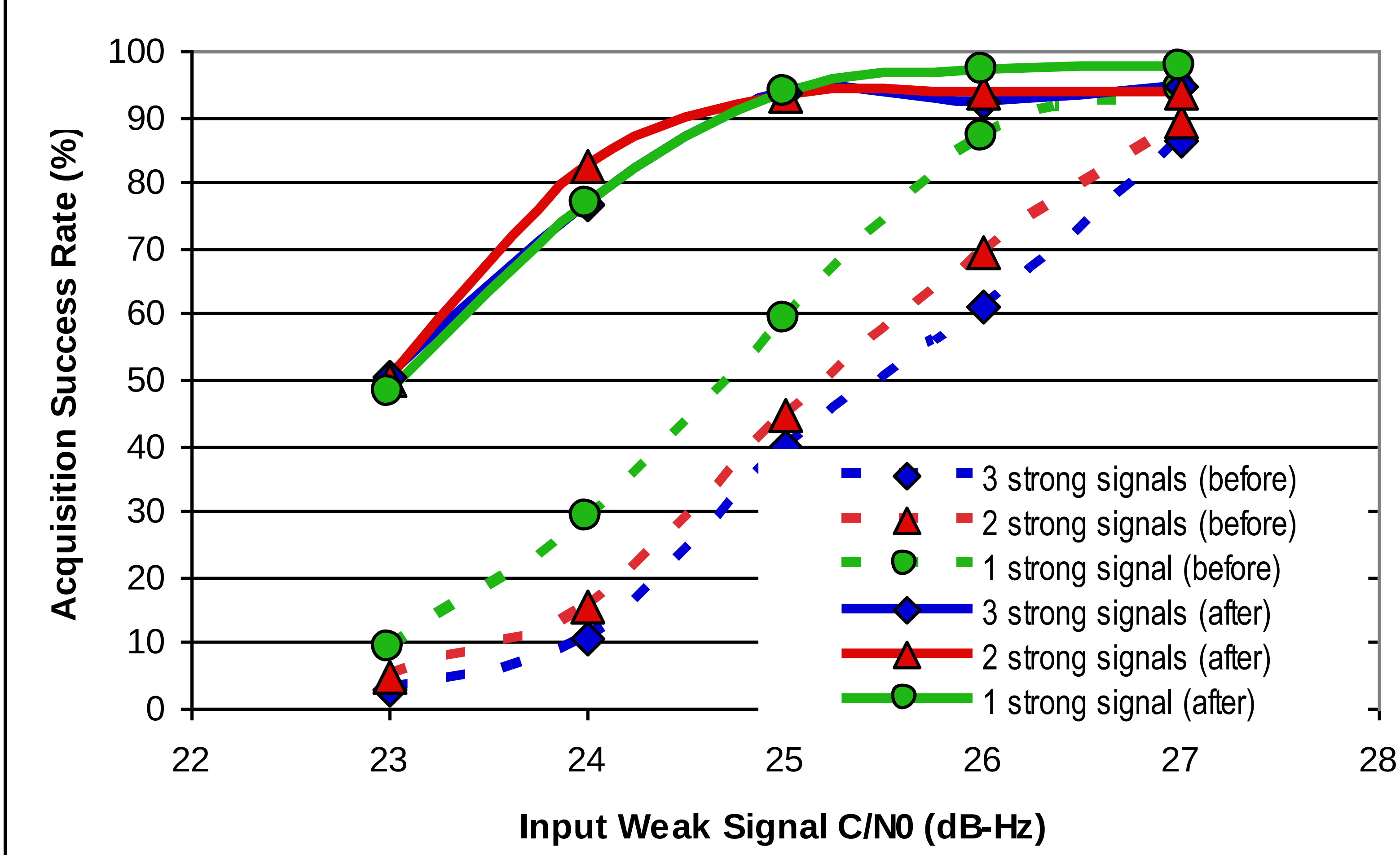


Weak Signal Acquisition Simulation Results



Weak Signal Acquisition Simulation Results

Acquisition successful rate before and after removal of strong signals (Strong signals C/N0 =44 dB-Hz).



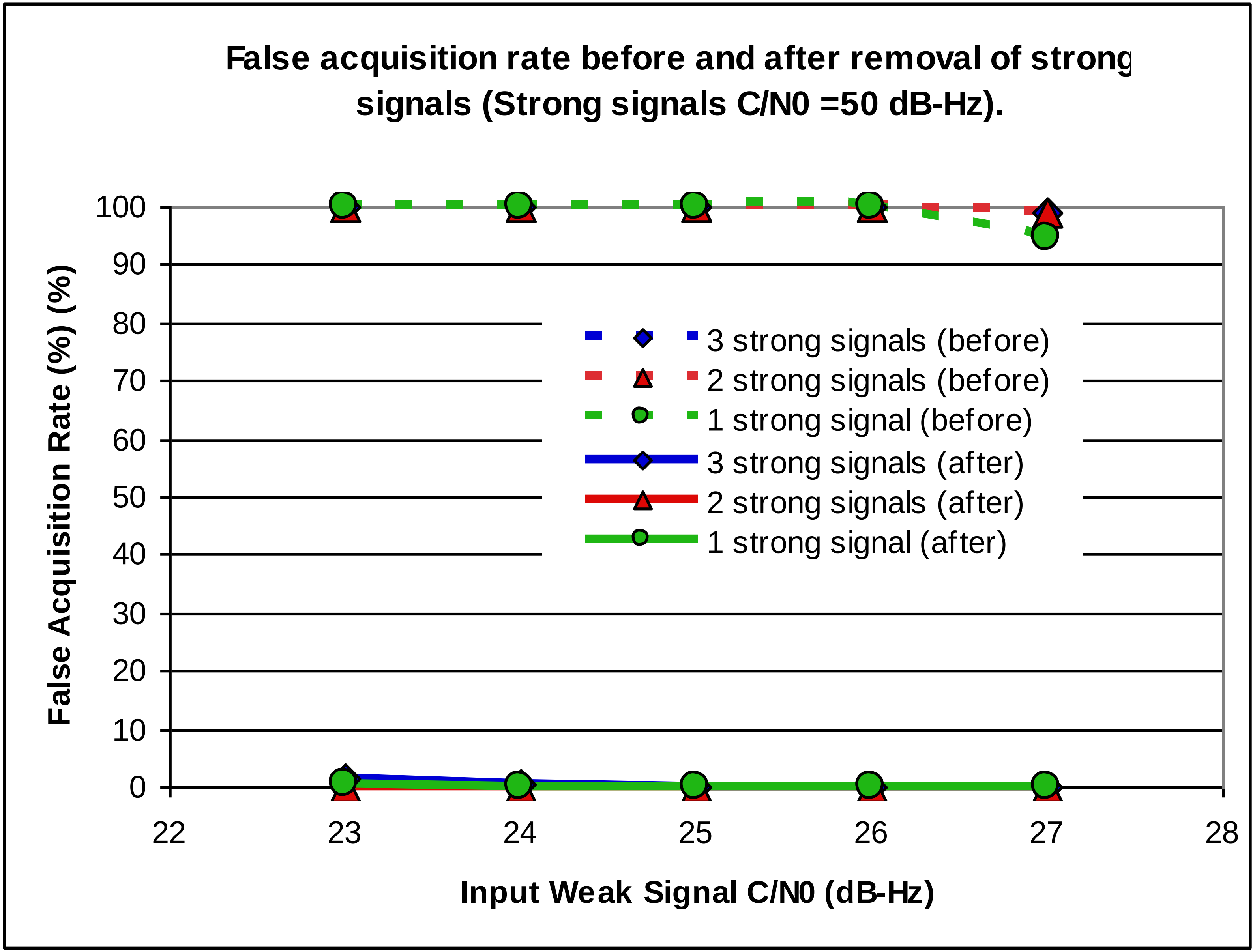
Weak Signal Acquisition Simulation Results

Problem Statement

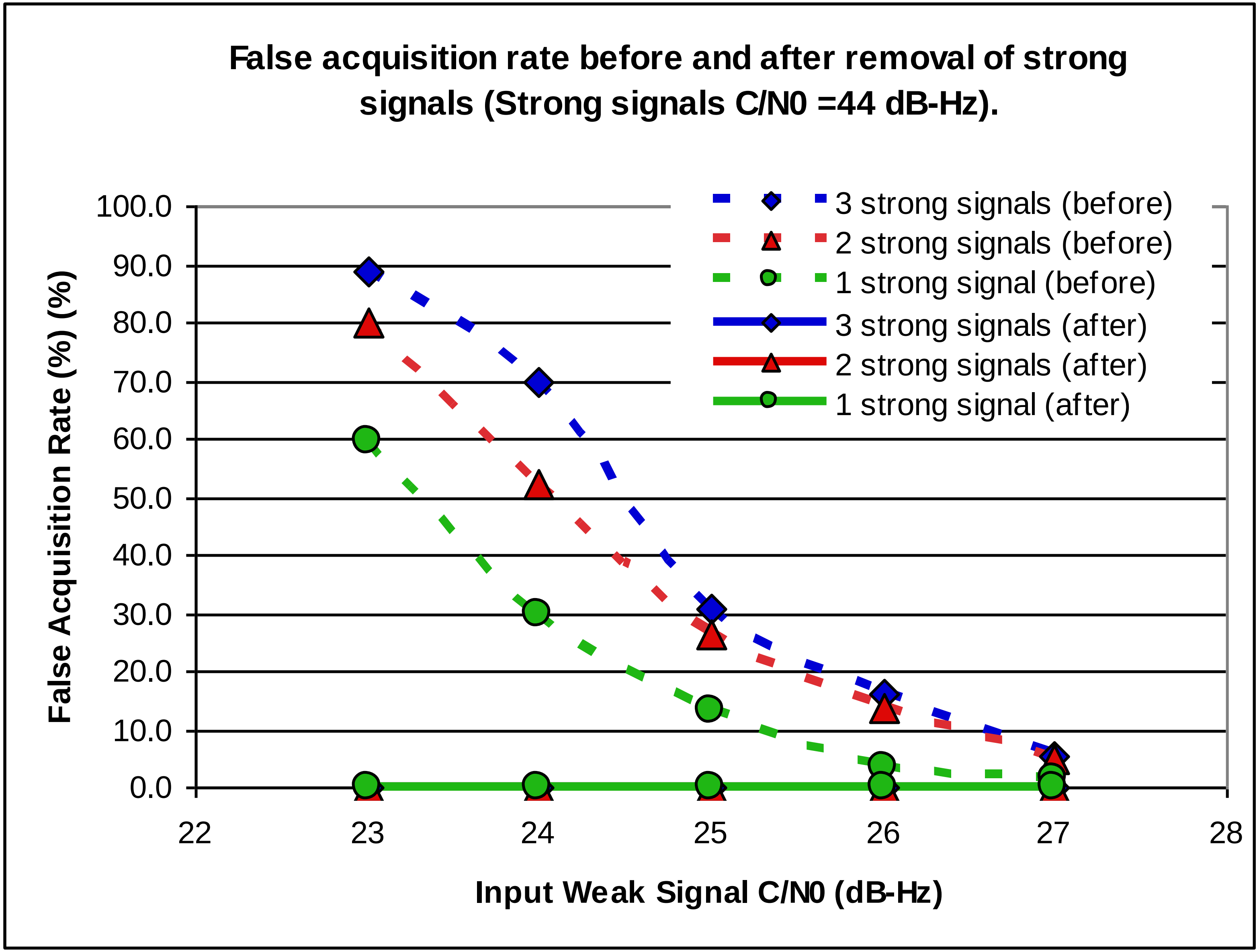
Self-Interference Assessment

Subspace Projection Method

Simulation Results


False acquisition rate before and after removal of strong signals (Strong signals C/N0 =50 dB-Hz).

Weak Signal Acquisition Simulation Results



False acquisition rate before and after removal of strong signals (Strong signals C/N0 =44 dB-Hz).

Conclusions

- Self interference is an important issue that needs to be addressed for a variety of applications
- Removing interference is a viable means to mitigate the self-interference during acquisition
- Partitioned subspace projection can effectively mitigate self-interference from multiple strong signals
- Computational expense reasonable