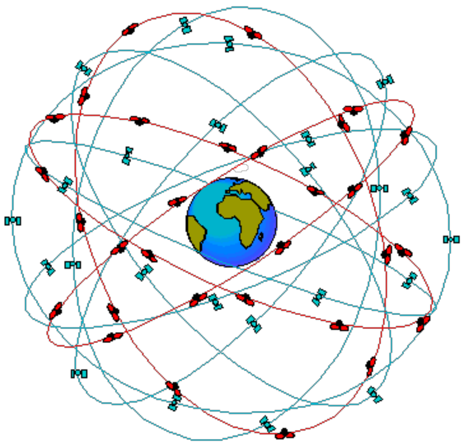
# Antenna Phase Center Variation in Multipath Environment



***Waldemar Kunysz***

***NovAtel Inc.***

***Email: waldemar.kunysz@novatel.com***

# Phase Center – where it matters?

- **Ranging applications (positioning, attitude, surveying, etc.)**
- **Imaging applications (radio-telescope, image radars)**
- **Antenna array applications (i.e. null steering)**
- **UWB (signal cohesion and group delay distortion)**

NovAtel
Precise thinking

# IEEE Standard Phase Centre Definition

**The location of a point associated with an antenna such that, if it is taken as the center of a sphere whose radius extends into the far-field, the phase of a given field component over the surface of the radiation sphere is "essentially" constant, at least over the portion of the surface where the radiation is significant.**

**Notes:** Some antennas do not have a unique phase center.

Far-Field condition: $\Delta\psi \leq 22.5^{\circ} \Rightarrow (\lambda_o/8)$

**Source:** Antenna Standards Committee of the IEEE Antennas and Propagation Group

Precise thinking

# Phase Center Measurements Methods

- **Second Derivative Method [1]**
- **Two Point Method [2]**
- **Edge Diffraction Method [3]**
- **Differential Phase Method [4]**
- **Three Antenna Method [5]**
- **Intuitive for "dummies" Method (this presentation)**
- **Time of Arrival (TOA) Method [6]**

NovAtel

Precise thinking

# Phase Center Model

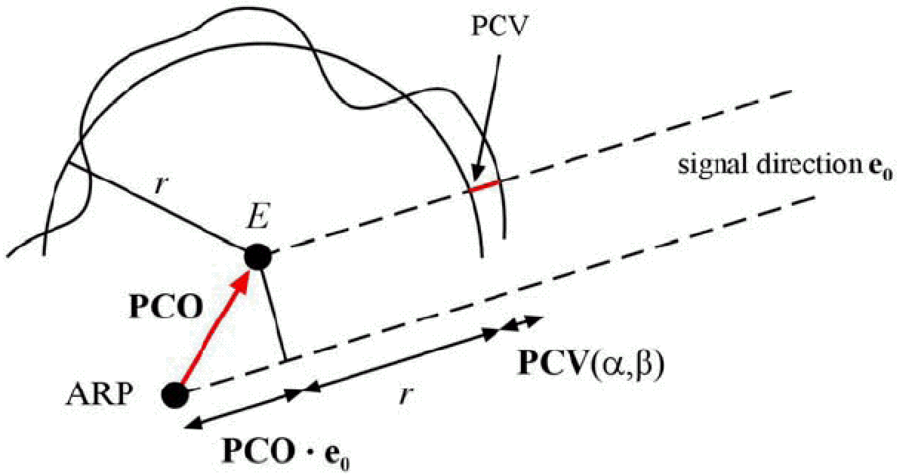



**Fig. 1**: Antenna model (according to *Zeimetz and Kuhlmann* 2006).

$e_0$ - is the unit vector of satellite obse$_0$ - is the unit vector of satellite observation (From sat. to PCO),

r - is the radius of perfect phase sphere originating at E

$$Correction\ Function: \quad S_{ARP} = r + PCO \cdot e_0 + PCV(\alpha, \beta) + \varepsilon \qquad (1)$$

PCO is found by minimizing the cost function:

$$\sum (PCV)^2 = Min$$

# Phase Center Determination

$$Phase(\phi,\theta) = \frac{2\pi}{\lambda}\left(x \cdot \cos\phi \sin\theta + y \cdot \sin\phi \sin\theta + z \cdot \cos\theta\right) + d$$

*or in matrix form:*

$$F(\phi,\theta) = \frac{2\pi}{\lambda}\begin{bmatrix}\cos\phi\sin\theta & \sin\phi\sin\theta & \cos\theta & 1\end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ d' \end{bmatrix} \quad => \quad F = M \cdot P \quad (1)$$

Precise thinking

# Phase Center Determination

$$Phase(\phi,\theta) = \frac{2\pi}{\lambda}\left(x\cdot\cos\phi\sin\theta + y\cdot\sin\phi\sin\theta + z\cdot\cos\theta\right) + d$$

*or in matrix form:*

$$F(\phi,\theta) = \frac{2\pi}{\lambda}\begin{bmatrix}\cos\phi\sin\theta & \sin\phi\sin\theta & \cos\theta & 1\end{bmatrix}\cdot\begin{bmatrix}x\\ y\\ z\\ d'\end{bmatrix} \quad => \quad F = M\cdot P \quad (1)$$

*To find phase center P we use eq.* (1), *hence:*

$$P = M^{-1}\cdot F$$

$$\begin{bmatrix}x\\ y\\ z\\ d'\end{bmatrix} = \frac{\lambda}{2\pi}\begin{bmatrix}\cos\phi\sin\theta & \sin\phi\sin\theta & \cos\theta & 1\end{bmatrix}^{-1}\cdot F(\phi,\theta)$$

NovAtel

Precise thinking

# Assemble solution matrix

$$\begin{bmatrix} x^1 & x^2 & : & x^L \\ y^1 & y^2 & : & y^L \\ z^1 & y^2 & : & y^L \\ d^1 & d^2 & : & d^L \end{bmatrix} = \frac{1}{2\pi} \bullet \begin{bmatrix} \cos\phi_1 \sin\theta_1 & \sin\phi_1 \sin\theta_1 & \cos\theta_1 & 1 \\ \cos\phi_2 \sin\theta_1 & \sin\phi_2 \sin\theta_1 & \cos\theta_1 & 1 \\ : & : & : & : \\ \cos\phi_N \sin\theta_1 & \sin\phi_N \sin\theta_1 & \cos\theta_1 & 1 \\ \cos\phi_1 \sin\theta_2 & \sin\phi_1 \sin\theta_2 & \cos\theta_2 & 1 \\ : & : & : & : \\ \cos\phi_N \sin\theta_K & \sin\phi_N \sin\theta_K & \cos\theta_K & 1 \end{bmatrix}^{-1} \bullet \begin{bmatrix} \varphi_1^1(\phi_1,\theta_1)\cdot\lambda^1 & \varphi_1^2(\phi_1,\theta_1)\cdot\lambda^2 & : & \varphi_1^L(\phi_1,\theta_1)\cdot\lambda^L \\ \varphi_2^1(\phi_2,\theta_1)\cdot\lambda^1 & \varphi_1^2(\phi_2,\theta_1)\cdot\lambda^2 & : & \varphi_1^L(\phi_2,\theta_1)\cdot\lambda^L \\ : & : & : & : \\ \varphi_N^1(\phi_N,\theta_1)\cdot\lambda^1 & \varphi_1^2(\phi_N,\theta_1)\cdot\lambda^2 & : & \varphi_1^L(\phi_N,\theta_1)\cdot\lambda^L \\ \varphi_{N+1}^1(\phi_1,\theta_2)\cdot\lambda^1 & \varphi_{N+1}^2(\phi_1,\theta_2)\cdot\lambda^2 & : & \varphi_{N+1}^L(\phi_1,\theta_2)\cdot\lambda^L \\ : & : & : & : \\ \varphi_{N\cdot K}^1(\phi_N,\theta_K)\cdot\lambda^1 & \varphi_{N\cdot K}^2(\phi_N,\theta_K)\cdot\lambda^2 & : & \varphi_{N\cdot K}^L(\phi_N,\theta_K)\cdot\lambda^L \end{bmatrix}$$

$$where: \quad \varphi_i = 0 : \frac{2\pi}{N} : 2\pi, \quad \theta_i = 0 : \frac{\pi}{2K} : \frac{\pi}{2} \quad (upper \quad hemisphere)$$

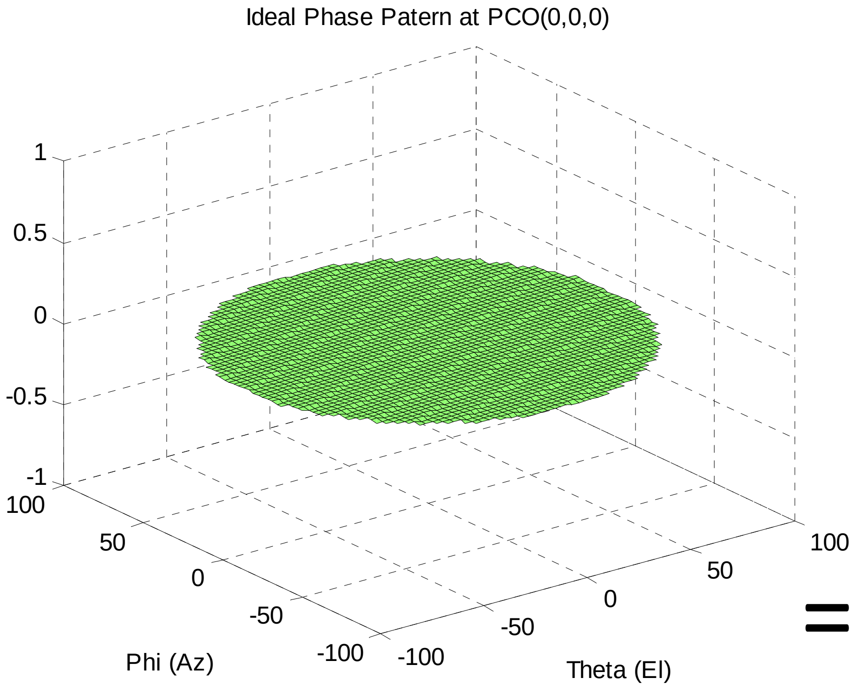
$$or: \quad \varphi_i = -\frac{\pi}{2} : \frac{\pi}{N} : \frac{\pi}{2}, \quad \theta_i = -\frac{\pi}{2} : \frac{\pi}{K} : \frac{\pi}{2} \quad (upper \quad hemisphere)$$

$L:$ *Number of frequencies*

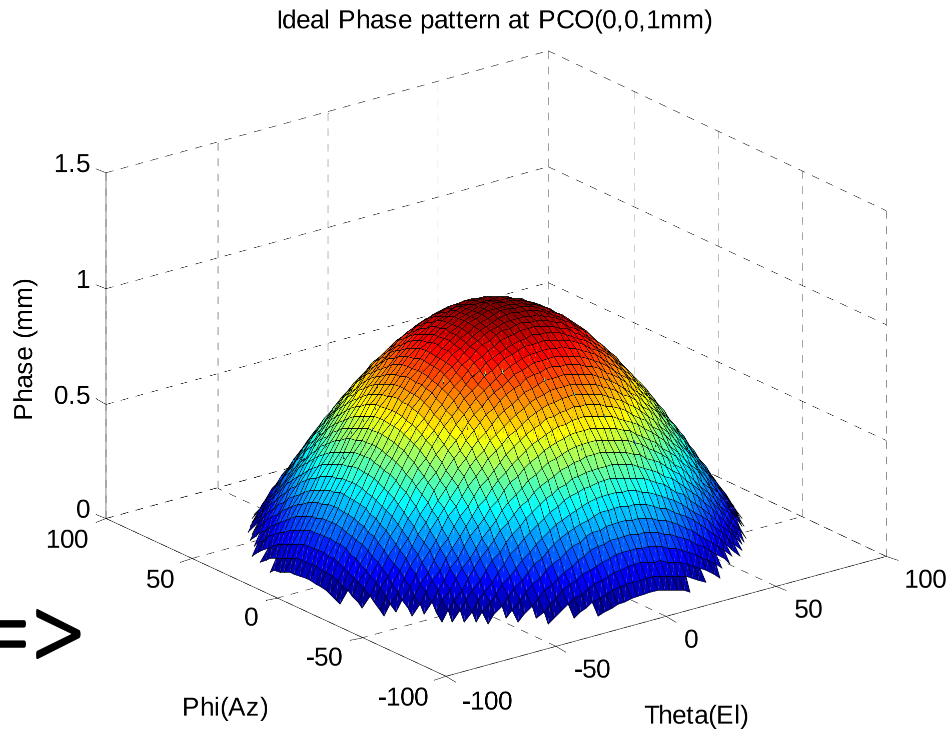
NovAtel

Precise thinking

# Ideal Phase Patterns

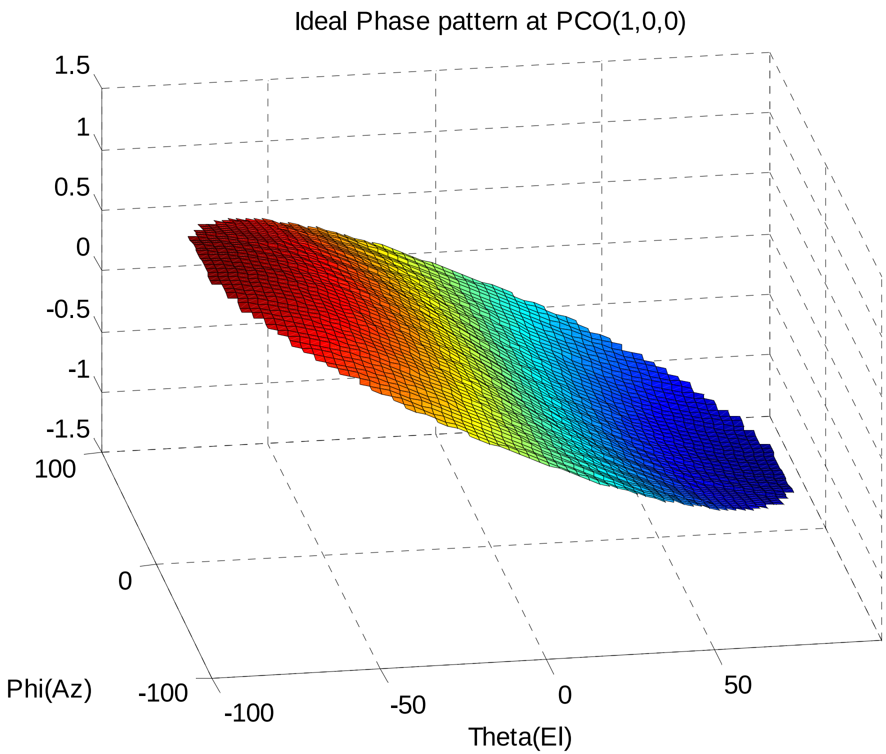


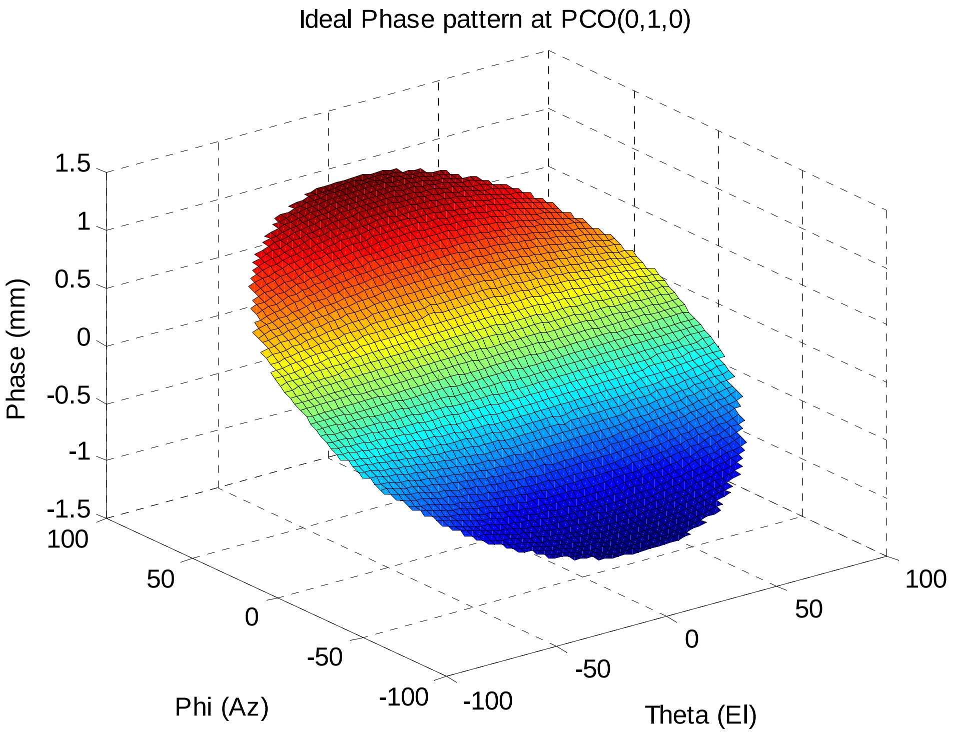

==>




Ideal Phase Pattern shifted along Z-axis from origin (0,0,0) to (0,0,1)

# Shifted Ideal Phase Patterns



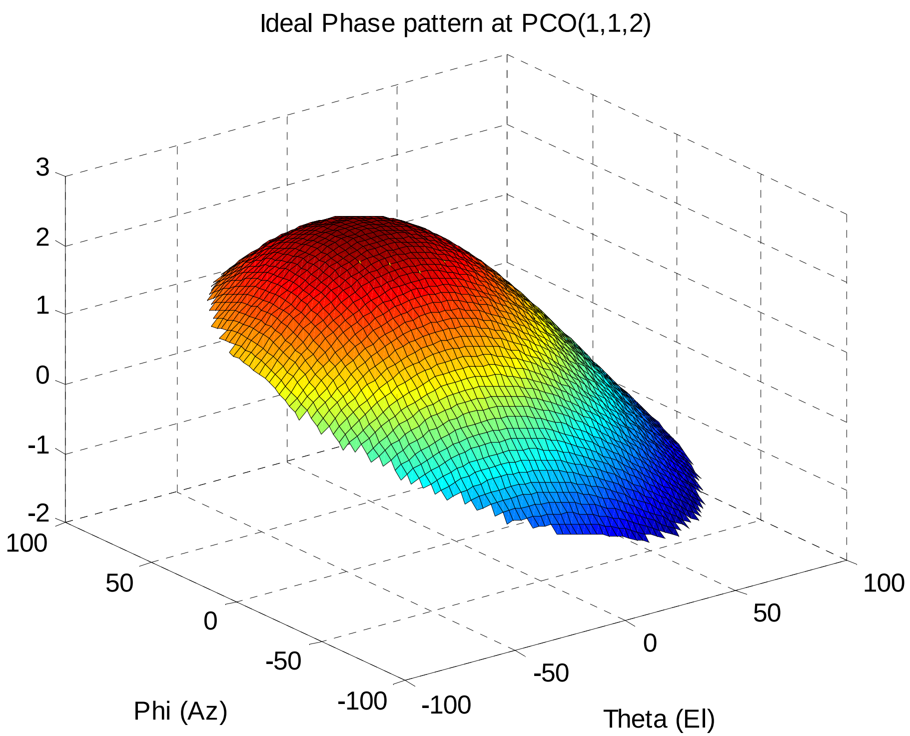




Ideal Phase Pattern shifted along X-axis from origin (0,0,0) to (1,0,0)

Ideal Phase Pattern shifted along Y-axis from origin (0,0,0) to (0,1,0)

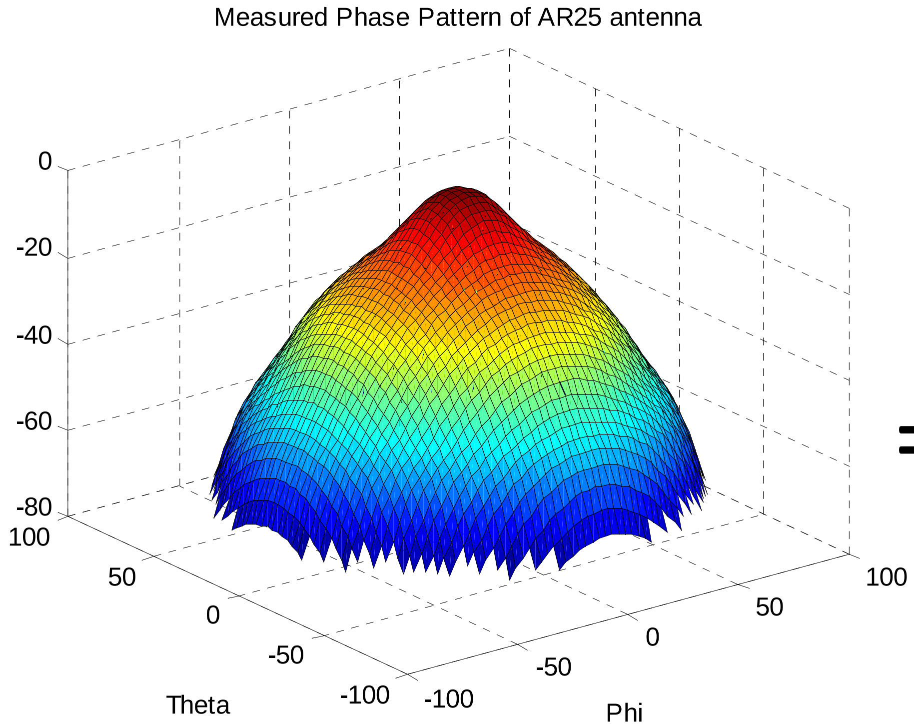
Precise thinking

# Shifted Ideal Phase Patterns




Ideal Phase Pattern shifted along X,Y,Z-axis from origin (0,0,0) to (1,1,2)

# Step 1 – Determine PCO

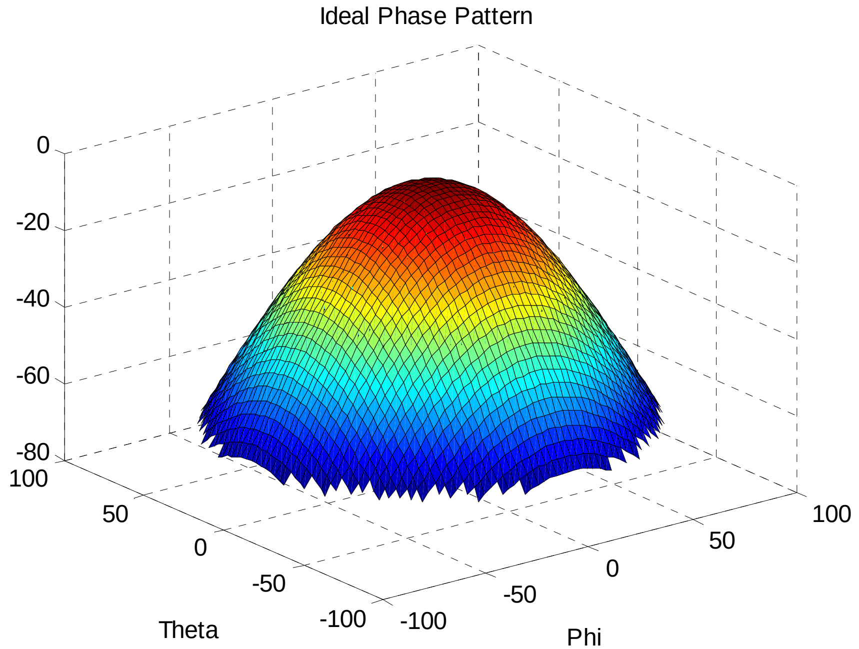



==> Compute mean PCO(x,y,z) using Least Square Method

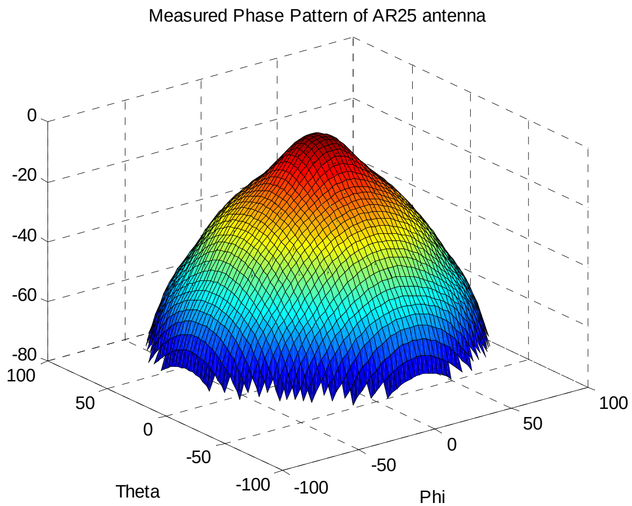
Measured phase pattern of AR25 antenna (note that this phase pattern shape is heavily weighted by large Z-axis offset of yet to be determined PCO value)

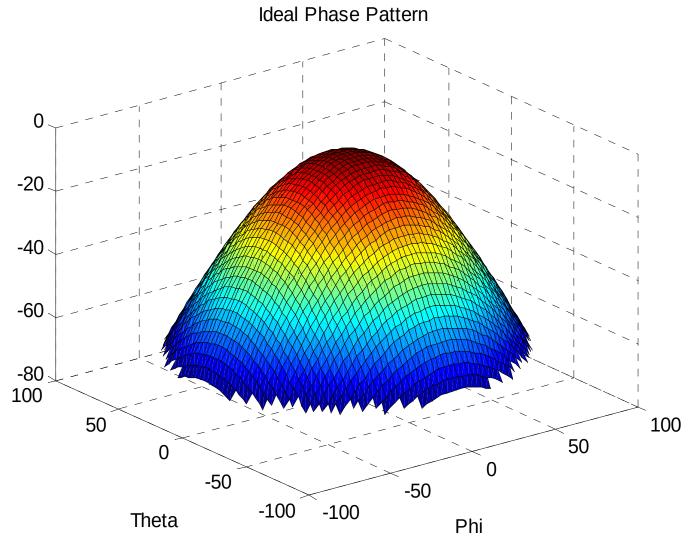
NovAtel Precise thinking

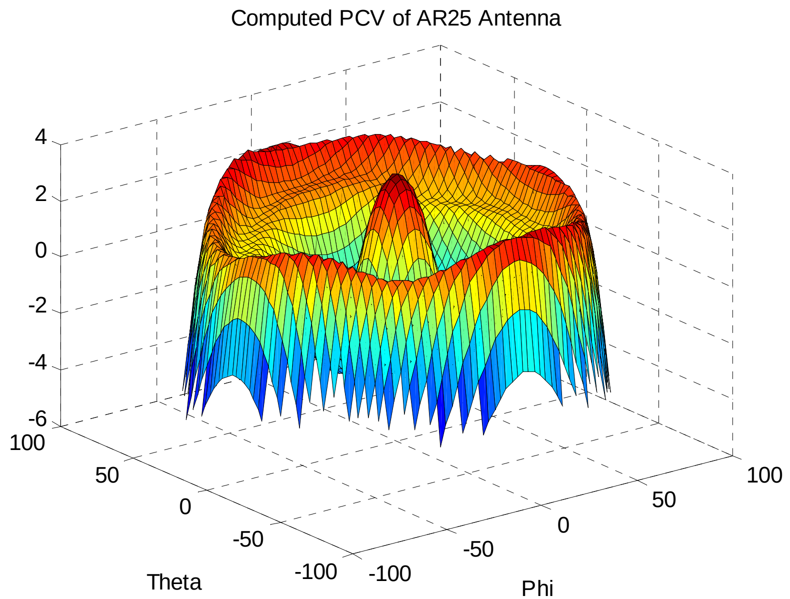
# Step 2 – Determine ideal spherical phase pattern at determined PCO location



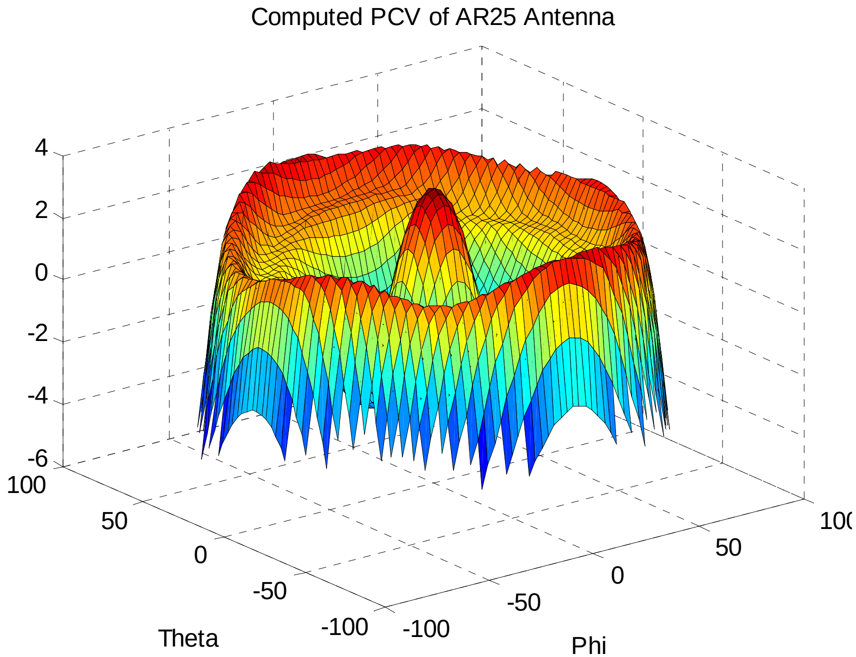
# Step 3 – Determine PCV

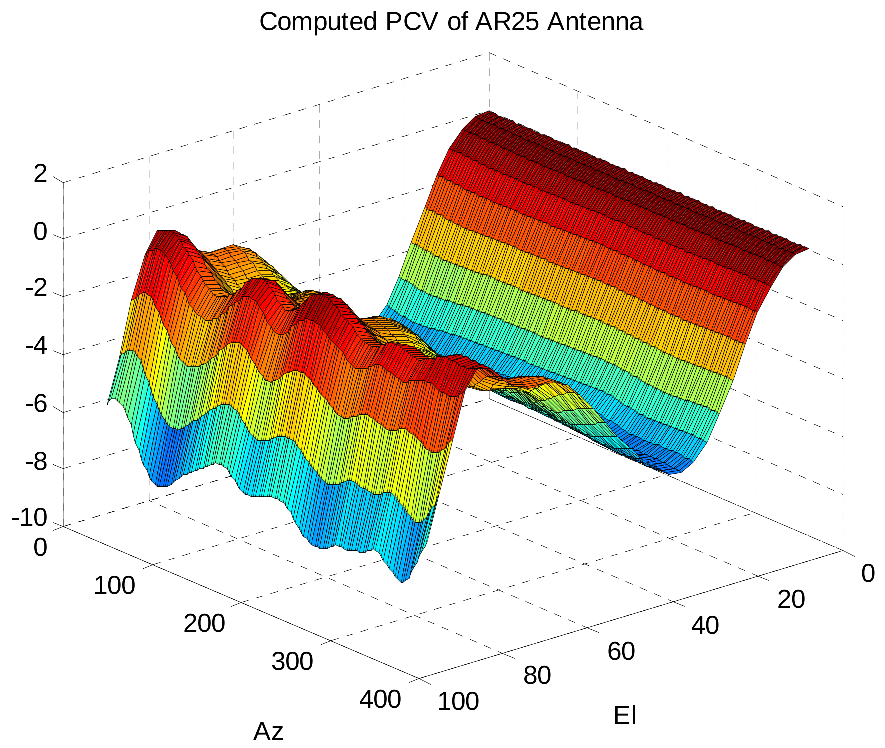


# Computed PCV around PCO:

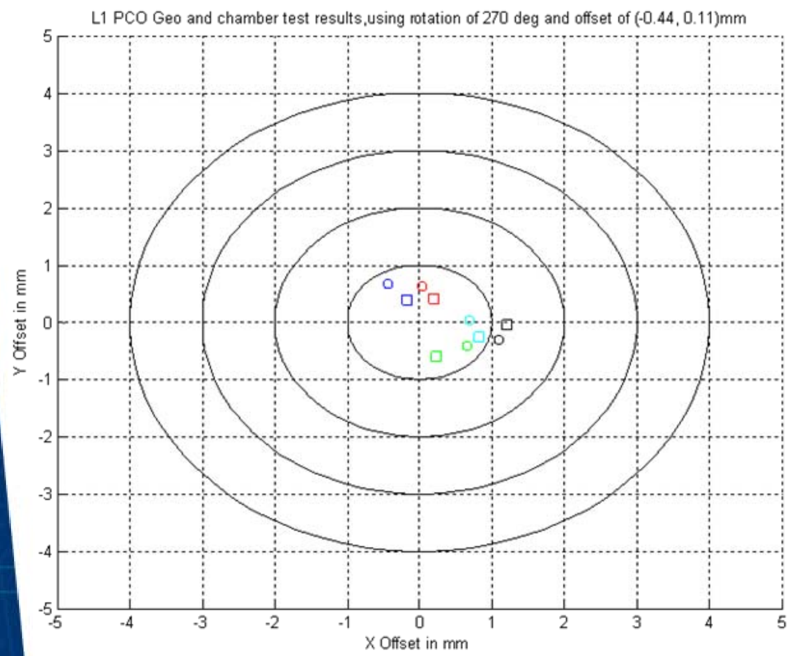


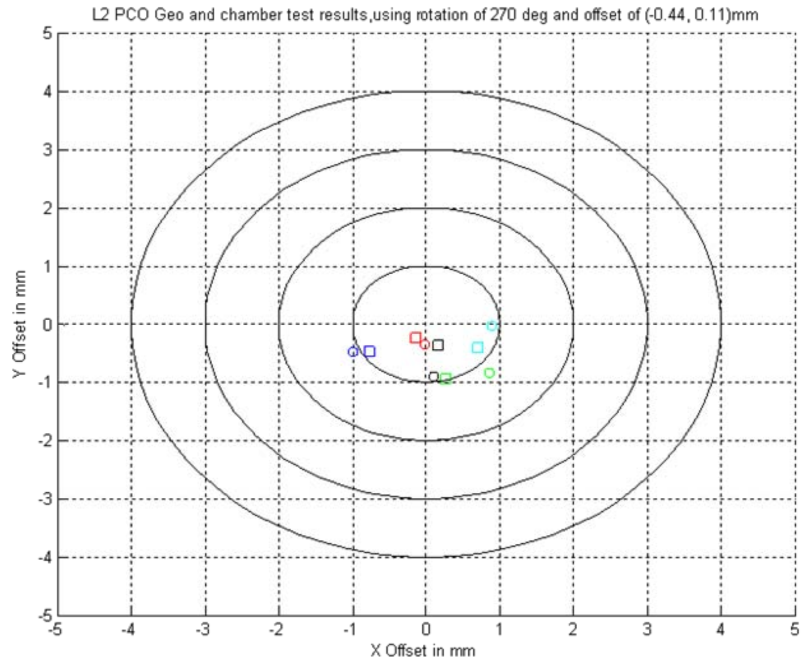




In

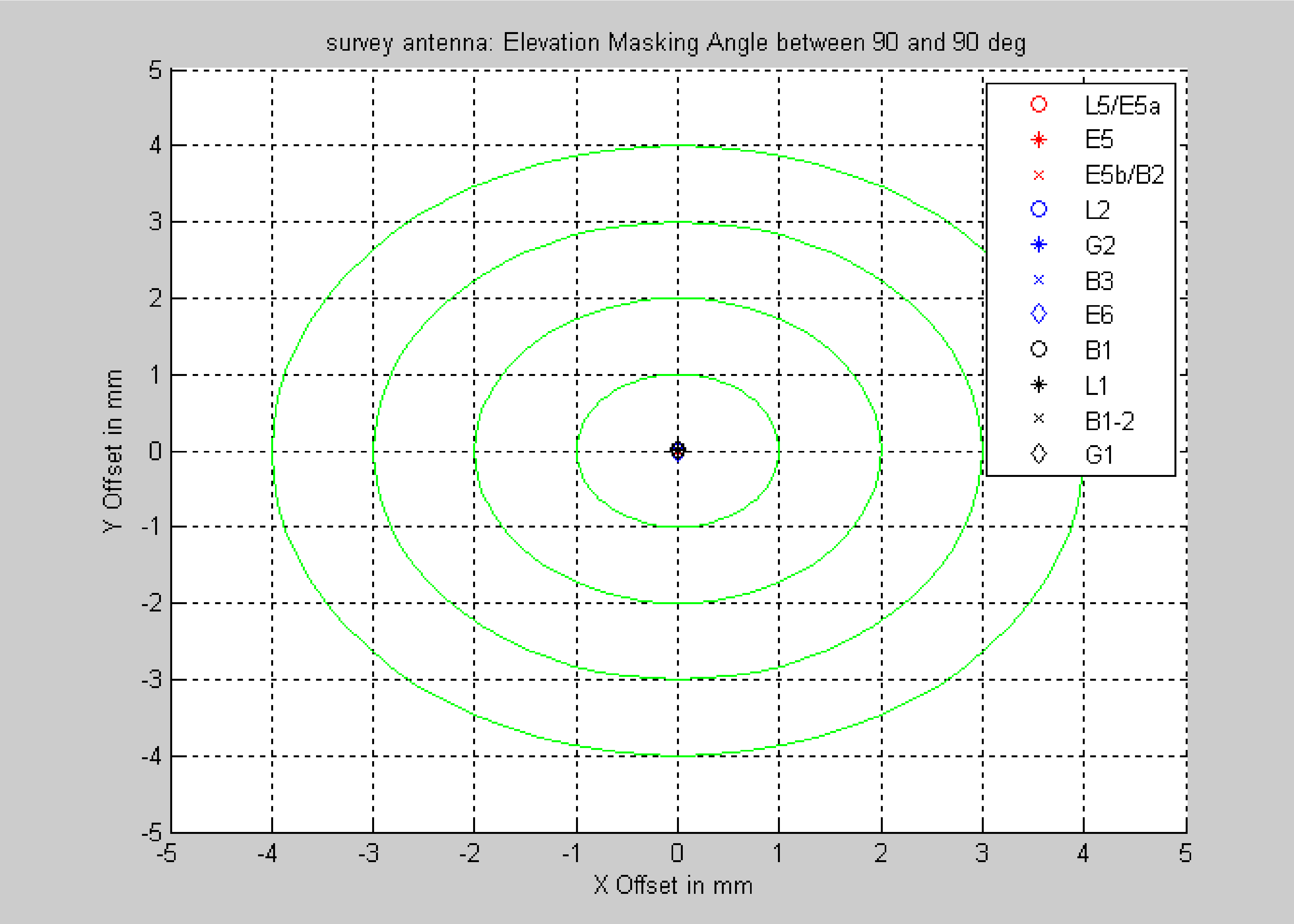
Precise thinking

# Example 1 – Anechoic chamber measurements versus Geo++ live GPS signal measurements



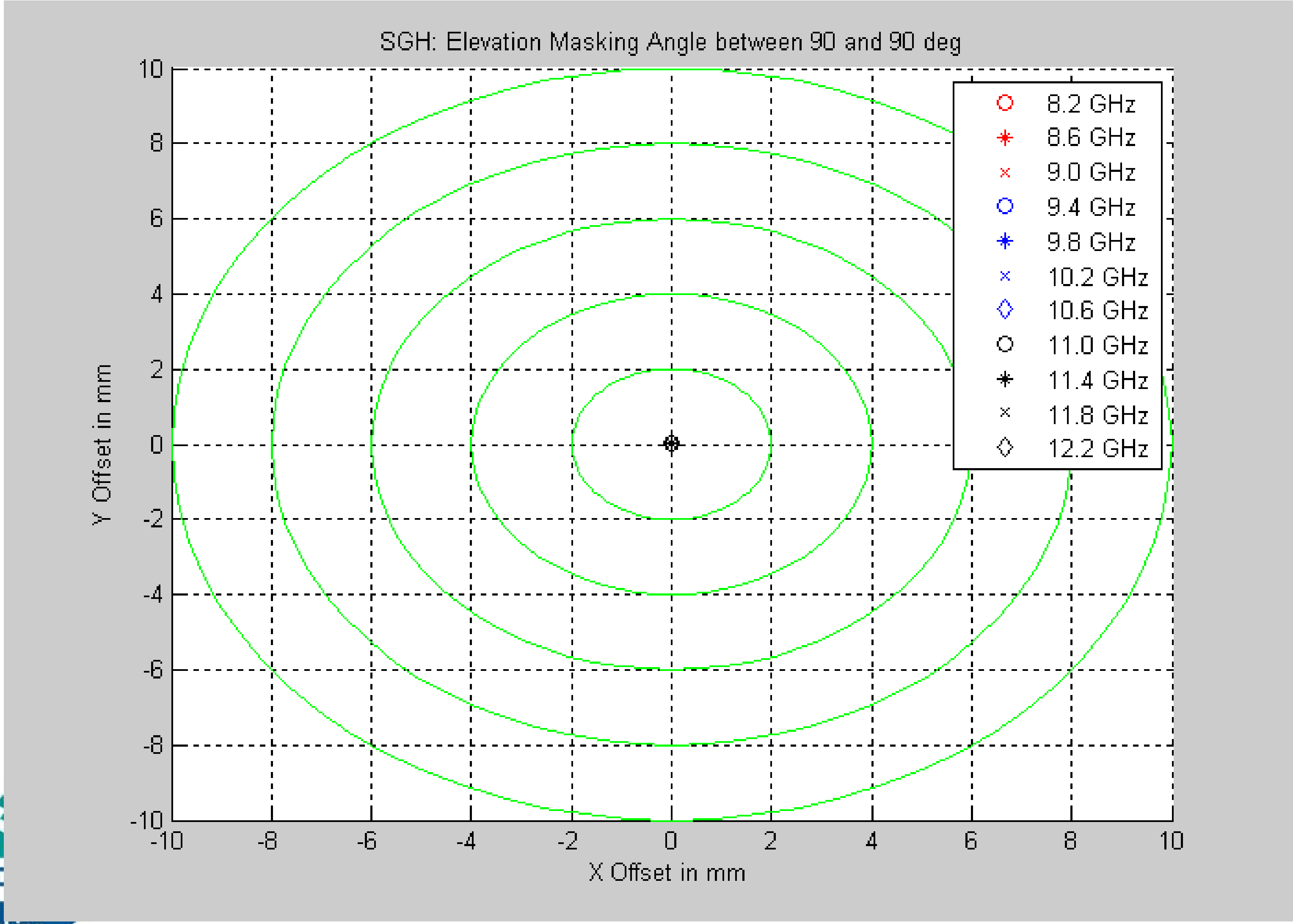




Precise thinking

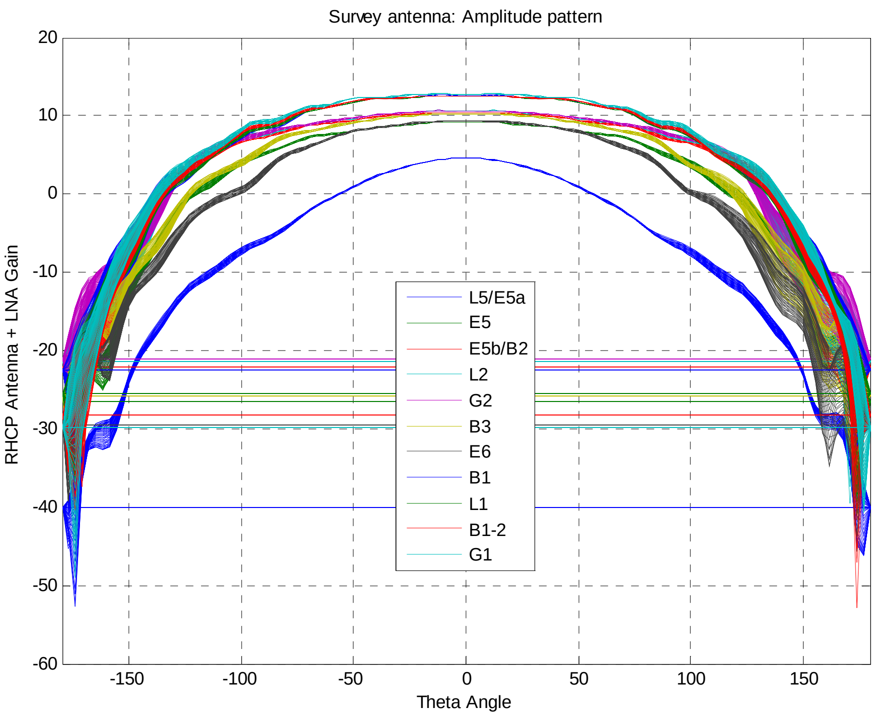


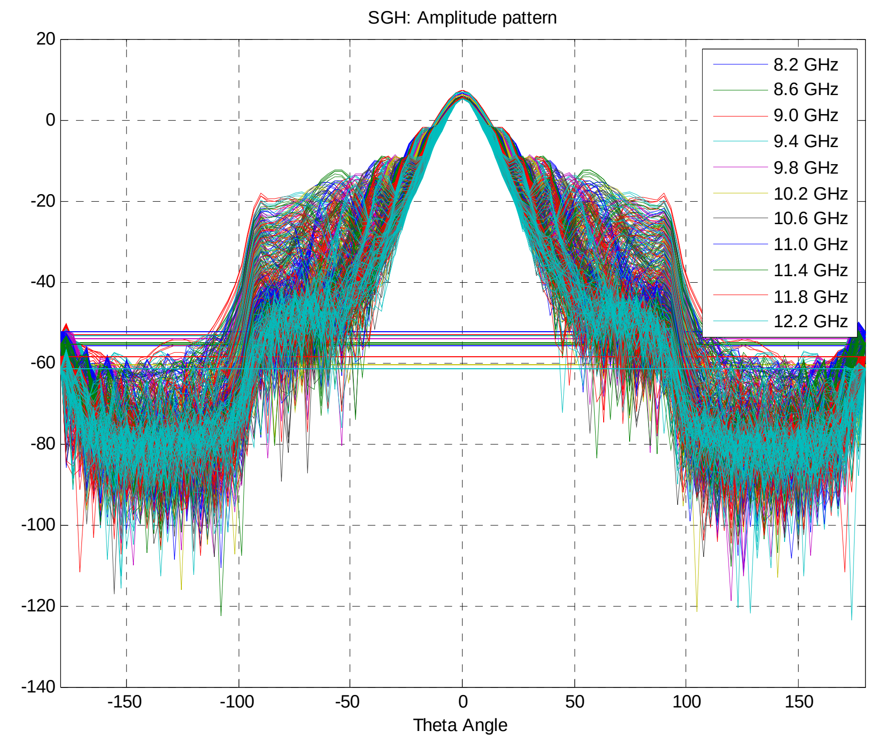

NovAtel
Precise thinking

# Horizontal Phase Center of SGH Antenna



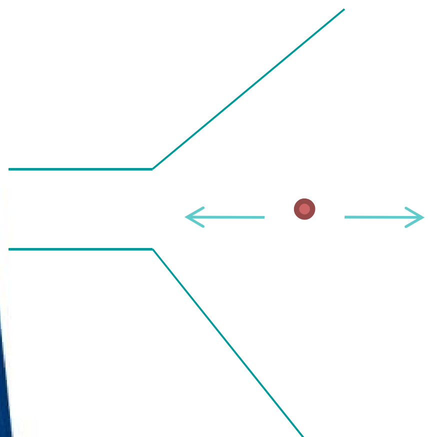
# Radiation Amplitude Patterns (GNSS and SGH Antenna)

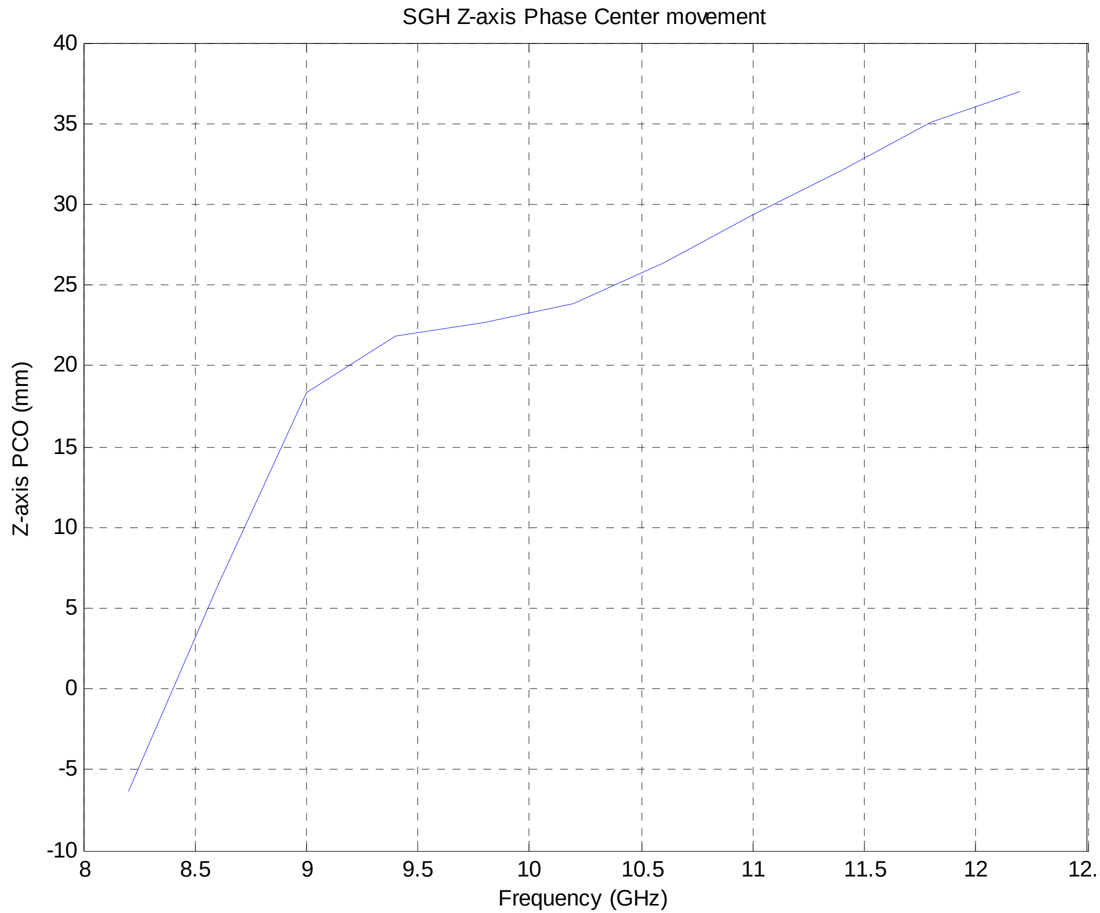



Precise thinking

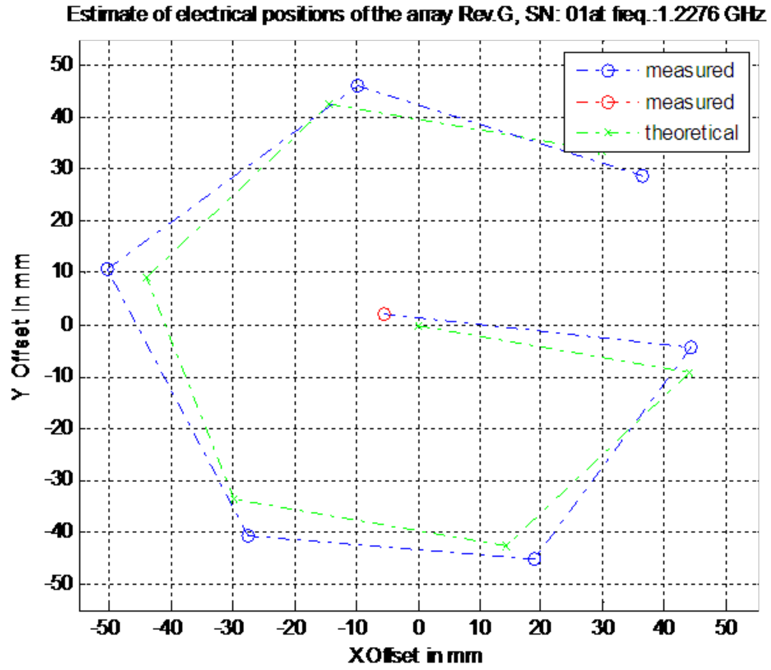
# PCO movement in Z-axis (SGH antenna)

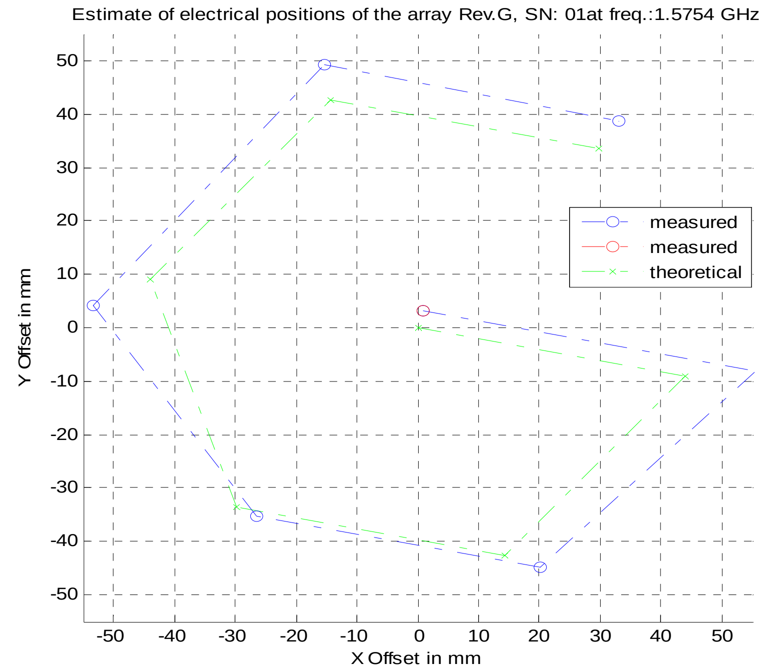




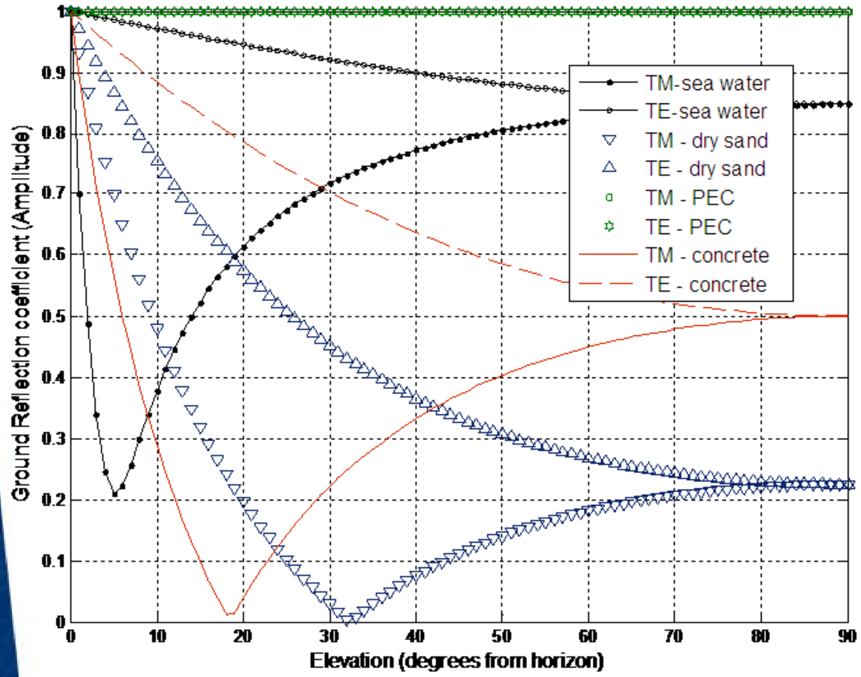

Precise thinking

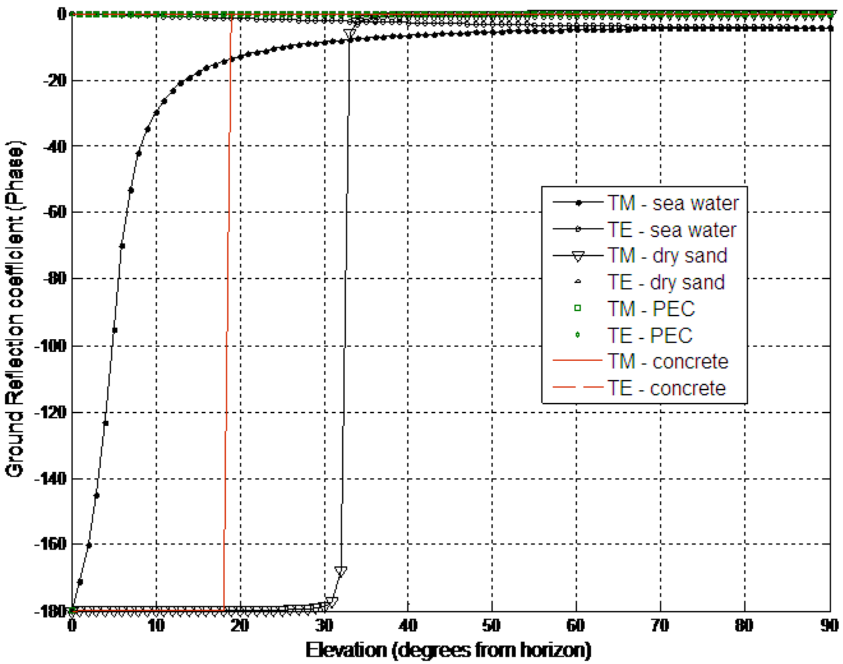
# Example 2 - Tightly coupled Array antenna



# 2-Ray Multipath Model using single reflector

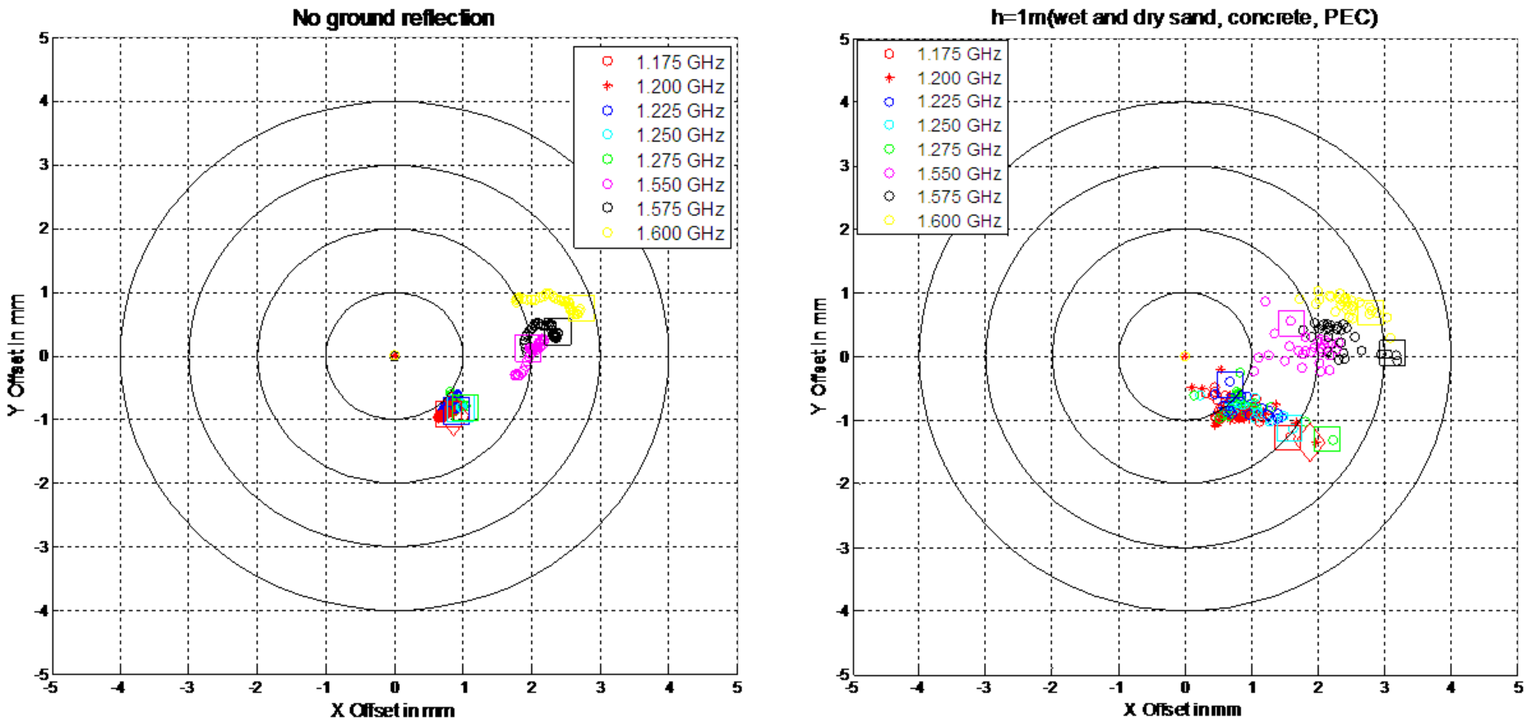


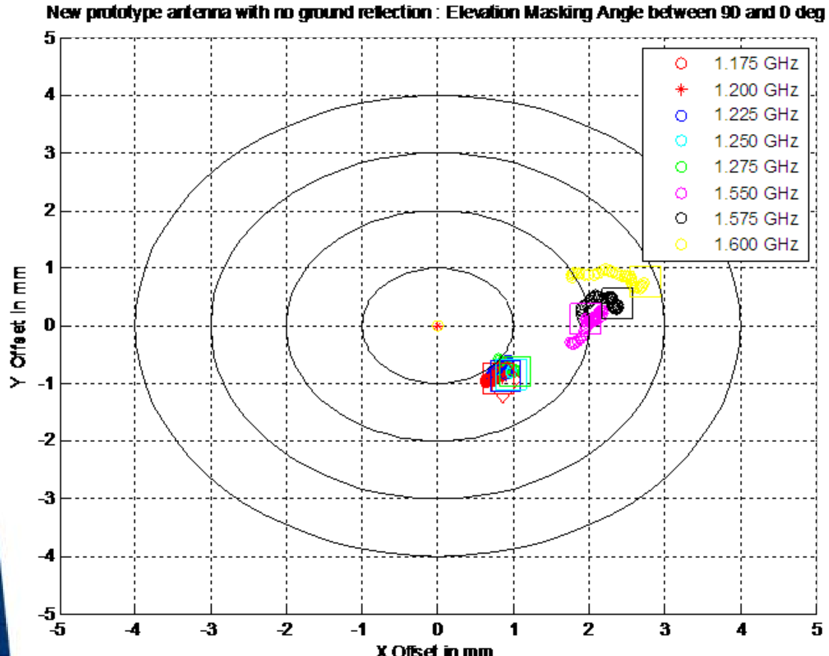

$$\Gamma^{TE} = \frac{\eta_2 \cos\phi^i - \eta_1 \cos\phi^t}{\eta_2 \cos\phi^i + \eta_1 \cos\phi^t}$$

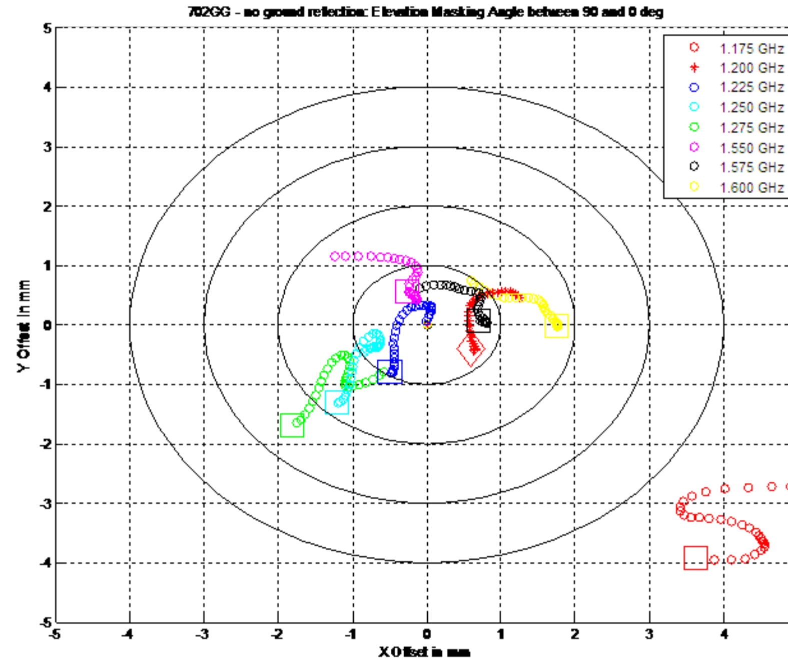
$$\Gamma^{TM} = \frac{\eta_2 \cos\phi^t - \eta_1 \cos\phi^i}{\eta_2 \cos\phi^t + \eta_1 \cos\phi^i}$$

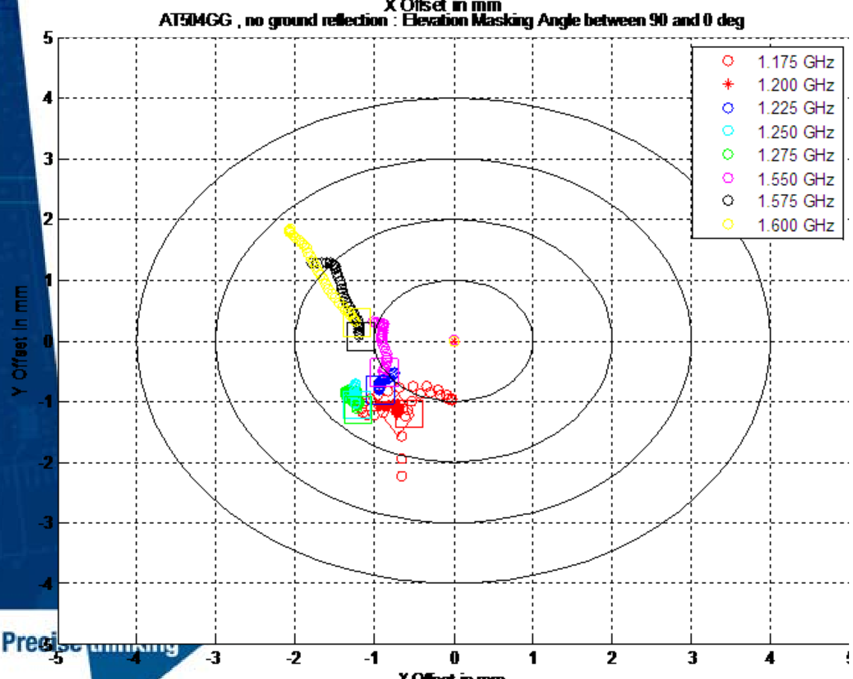
# Multipath Effect on PCO
# (Example of antenna with good multipath performance)

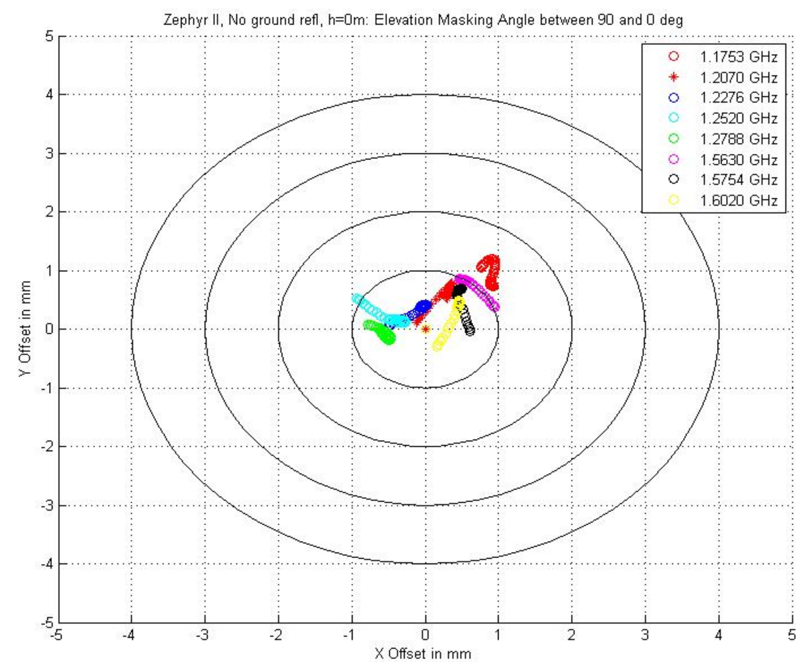


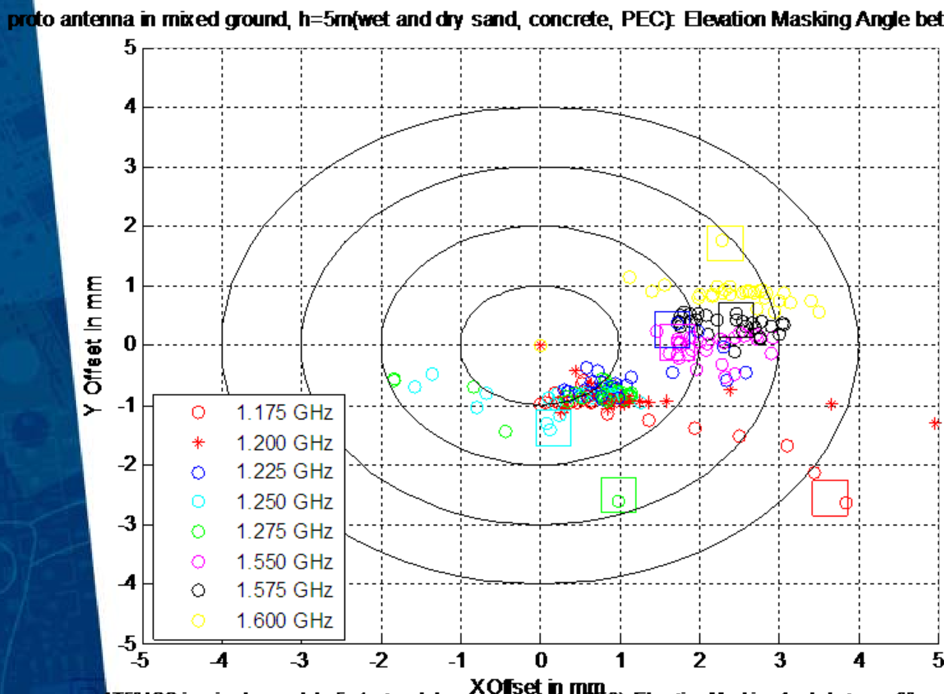

NovAtel
Precise thinking

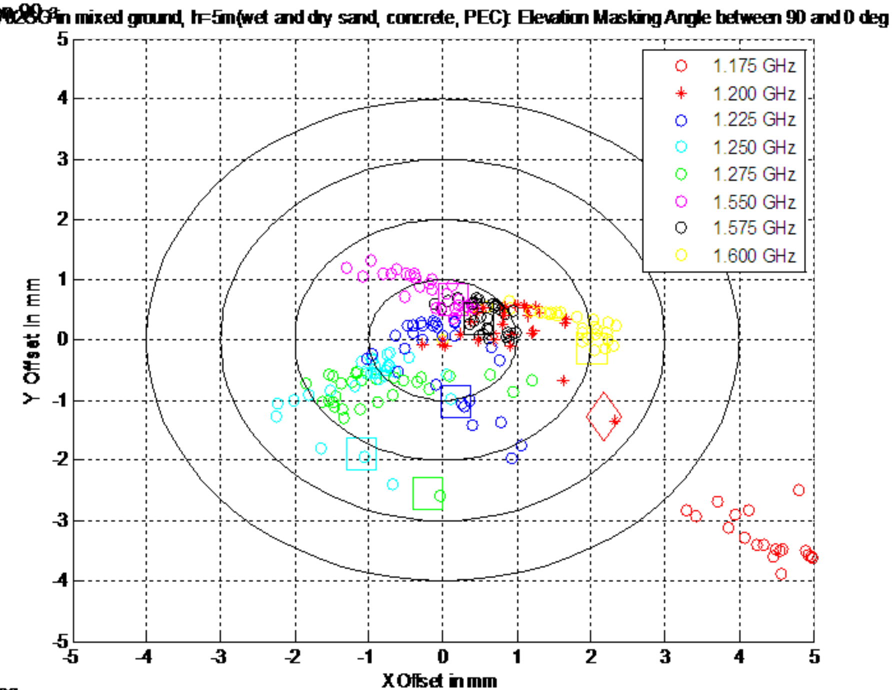

New prototype antenna with no ground reflection : Elevation Masking Angle between 90 and 0 deg
Y Offset in mm
X Offset in mm
1.175 GHz
1.200 GHz
1.225 GHz
1.250 GHz
1.275 GHz
1.550 GHz
1.575 GHz
1.600 GHz

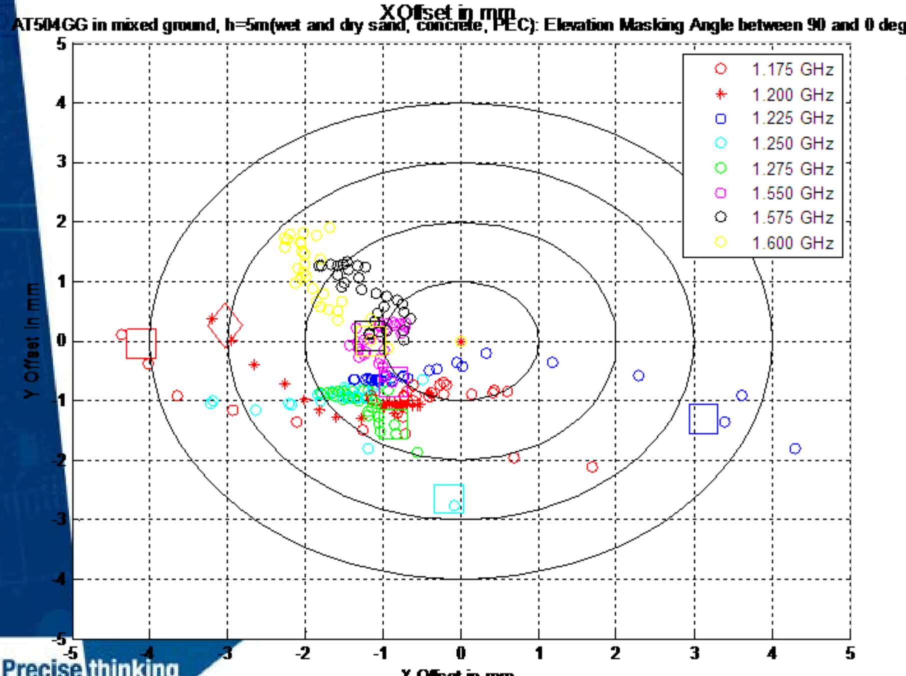

702GG - no ground reflection: Elevation Masking Angle between 90 and 0 deg
Y Offset in mm
X Offset in mm
1.175 GHz
1.200 GHz
1.225 GHz
1.250 GHz
1.275 GHz
1.550 GHz
1.575 GHz
1.600 GHz

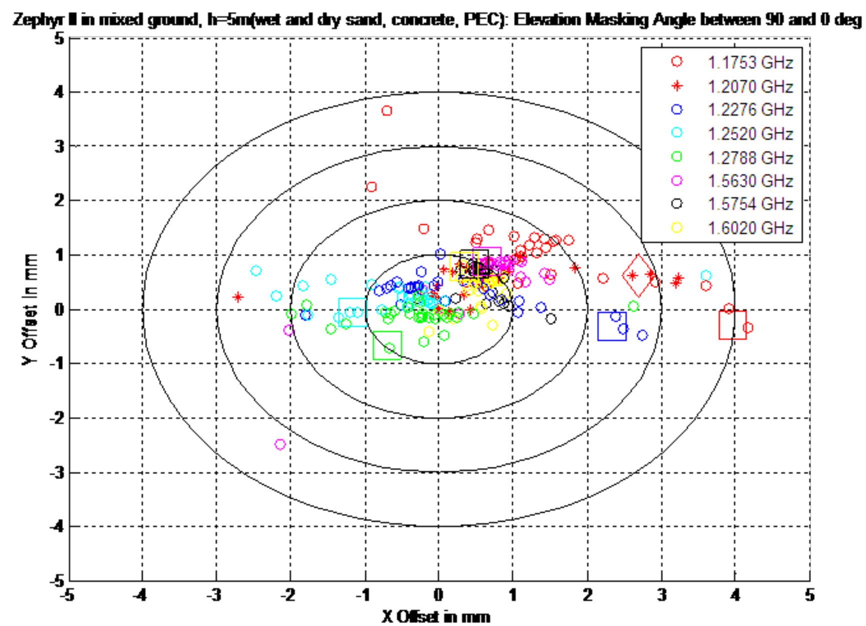

AT504GG , no ground reflection : Elevation Masking Angle between 90 and 0 deg
Y Offset in mm
X Offset in mm
1.175 GHz
1.200 GHz
1.225 GHz
1.250 GHz
1.275 GHz
1.550 GHz
1.575 GHz
1.600 GHz


Zephyr II, No ground refl, h=0m: Elevation Masking Angle between 90 and 0 deg
Y Offset in mm
X Offset in mm
1.1753 GHz
1.2070 GHz
1.2276 GHz
1.2520 GHz
1.2788 GHz
1.5630 GHz
1.5754 GHz
1.6020 GHz


proto antenna in mixed ground, h=5m(wet and dry sand, concrete, PEC): Elevation Masking Angle between 90
Y Offset in mm
X Offset in mm
1.175 GHz
1.200 GHz
1.225 GHz
1.250 GHz
1.275 GHz
1.550 GHz
1.575 GHz
1.600 GHz


in mixed ground, h=5m(wet and dry sand, concrete, PEC): Elevation Masking Angle between 90 and 0 deg
Y Offset in mm
X Offset in mm
1.175 GHz
1.200 GHz
1.225 GHz
1.250 GHz
1.275 GHz
1.550 GHz
1.575 GHz
1.600 GHz


AT504GG in mixed ground, h=5m(wet and dry sand, concrete, PEC): Elevation Masking Angle between 90 and 0 deg
Y Offset in mm
X Offset in mm
1.175 GHz
1.200 GHz
1.225 GHz
1.250 GHz
1.275 GHz
1.550 GHz
1.575 GHz
1.600 GHz


Zephyr II in mixed ground, h=5m(wet and dry sand, concrete, PEC): Elevation Masking Angle between 90 and 0 deg
Y Offset in mm
X Offset in mm
1.1753 GHz
1.2070 GHz
1.2276 GHz
1.2520 GHz
1.2788 GHz
1.5630 GHz
1.5754 GHz
1.6020 GHz

# Multipath mitigation

$$\hat{S}(\varphi,\theta) = \hat{C}^*(\varphi,\theta)\cdot\hat{E} + \hat{C}^*(\varphi,-\theta)\cdot\Gamma\cdot\hat{E}\cdot e^{j2\pi d/\lambda}$$

$$where: \quad \hat{E} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -j\end{pmatrix} \quad ;\hat{C}^*(\varphi,\theta) = \frac{a_{\phi,\theta}}{\sqrt{2}}\begin{pmatrix}1 & j\end{pmatrix}e^{j\varphi(\phi,\theta)}$$

$$\Gamma = \begin{bmatrix}\Gamma^{TE} & 0\\ 0 & \Gamma^{TM}\end{bmatrix} \quad ;\hat{C}^*(\varphi,-\theta) = \frac{a_{\phi,-\theta}}{\sqrt{2}}\begin{pmatrix}1 & \pm j\end{pmatrix}e^{j\varphi(\phi,-\theta)} \qquad d = 2\cdot h\sin(\theta)$$

$$\hat{S}^{RHCP} = \frac{a_{R(\theta)}e^{j\delta(\theta)}}{\sqrt{2}}\begin{bmatrix}1 & j\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -j\end{bmatrix}$$

$$\hat{S}^{LHCP} = \frac{a_{L(\theta)}e^{j\xi(\theta)}}{\sqrt{2}}\begin{bmatrix}1 & -j\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -j\end{bmatrix}$$

NovAtel
Precise thinking

# Multipath mitigation

$$\hat{S}^{RHCP} = \frac{a_{R(\theta)}e^{j\delta(\theta)}}{\sqrt{2}}\begin{bmatrix}1 & j\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -j\end{bmatrix} + \frac{a_{R(-\theta)}e^{j\delta(-\theta))}}{\sqrt{2}}\begin{bmatrix}1 & \pm j\end{bmatrix}\begin{bmatrix}\Gamma^{TE} & 0 \\ 0 & \Gamma^{TM}\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -j\end{bmatrix}e^{j2\pi d/\lambda}$$

$$\hat{S}^{LHCP} = \frac{a_{L(\theta)}e^{j\xi(\theta)}}{\sqrt{2}}\begin{bmatrix}1 & -j\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -j\end{bmatrix} + \frac{a_{L(-\theta)}e^{j\xi(-\theta))}}{\sqrt{2}}\begin{bmatrix}1 & \pm j\end{bmatrix}\begin{bmatrix}\Gamma^{TE} & 0 \\ 0 & \Gamma^{TM}\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -j\end{bmatrix}e^{j2\pi d/\lambda}$$

NovAtel

Precise thinking

# Multipath mitigation

$$\hat{S}^{RHCP} = \frac{a_{R(\theta)} e^{j\delta(\theta)}}{\sqrt{2}} \begin{bmatrix} 1 & j \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \end{bmatrix} + \frac{a_{R(-\theta)} e^{j\delta(-\theta))}}{\sqrt{2}} \begin{bmatrix} 1 & \pm j \end{bmatrix} \begin{bmatrix} \Gamma^{TE} & 0 \\ 0 & \Gamma^{TM} \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \end{bmatrix} e^{j 2\pi d/\lambda}$$

$$\hat{S}^{LHCP} = \frac{a_{L(\theta)} e^{j\xi(\theta)}}{\sqrt{2}} \begin{bmatrix} 1 & -j \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \end{bmatrix} + \frac{a_{L(-\theta)} e^{j\xi(-\theta))}}{\sqrt{2}} \begin{bmatrix} 1 & \pm j \end{bmatrix} \begin{bmatrix} \Gamma^{TE} & 0 \\ 0 & \Gamma^{TM} \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \end{bmatrix} e^{j 2\pi d/\lambda}$$

$$\hat{S}^{RHCP} = a_{R(\theta)} e^{j\delta(\theta)} + \frac{a_{R(-\theta)} e^{j(2\pi d/\lambda + \delta(-\theta))}}{2} \left( \Gamma^{TE} \pm \Gamma^{TM} \right)$$

$$\hat{S}^{LHCP} = 0 + \frac{a_{L(-\theta)} e^{j(2\pi d/\lambda + \xi(-\theta))}}{2} \left( \Gamma^{TE} \pm \Gamma^{TM} \right)$$

$$\hat{S}^{LHCP} = a_{L'(\theta)} e^{j\xi(\theta)} + \frac{a_{L(-\theta)} e^{j(2\pi d/\lambda + \xi(-\theta))}}{2} \left( \Gamma^{TE} \pm \Gamma^{TM} \right) \quad where \quad a_{L'(\theta)} << a_{L(\theta)}$$

NovAtel

Precise thinking

# Multipath mitigation

$$Scaling\ \ Correction(\phi,\theta) = \frac{a_{R(-\theta)}e^{j\left[\delta(-\theta)-\xi(-\theta)\right]}}{a_{L(-\theta)}}$$

NovAtel

Precise thinking

# Conclusions & Final Remarks

- A simple method to determine Phase Center Offset/Location based on Least Square Method is described
- PCO can be very sensitive to near-by reflections surrounding an antenna
- Proper pattern control and controlling amount of x-pol is crucial to keep PCO stable
- Multipath mitigation can be employed using information  received from well designed dual polarized antenna

NovAtel

Precise thinking

# References


[1] D.Carter, “Phase Centers of microwave antennas”, IRE Trans. Antennas and Prop., Vol AP-4, pp 597-600, Oct. 1956

[2] Y.Y.Hu, “A method of determining phase centers and its applications to electromagnetic horns”, J. Franklin Inst., Vol.271, pp 31-39, Jan. 1961

[3] M. Teichman, “Determination of horn antenna phase centre by edge diffraction theory”, IEEE Trans. Aerospace Eelctr. Systems, Vol. AES-9, pp 875-882, Nov. 1973

[4]J. Thovinen, A. Lehto, A. Raisanenn, “Differential Phase Method”, Int. Symp. of Antennas and Propag. Society, Vol. 3, pp 1298-1301, Ma 7-11,1990

[5]T.W.Hertel,”Phase Center based on the three antenna method”, IEEE Int. Symp. of Antennas and Propag. Scoiety, Vol. 3, pp 816-819, June 22-27, 2003

[6]Y.Yashcheshyn,M.Bury,K.Kurek,P.Bajurko, ”Evaluation of the impact of the virtual phase centre effect on the accuracy of the positioning system”, 3rd European Conf. on Antennas and Prop., pp 2930-2933, March 23-27, 2009


NovAtel

Precise thinking