Precise GNSS positioning with Android smartphones and on the use of the best integer equivariant estimator

Part 1: Instantaneous cm-level RTK positioning using Google Pixel 4 and Samsung Galaxy S20 smartphones
Part 2: RTK positioning for some recent Android smartphones (Google Pixel5, Pixel6 Pro, Pixel7 Pro and Samsung Galaxy S22)

Dr Robert Odolinski

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Part 1: Instantaneous, dual-frequency, multi-GNSS precise cm-level RTK positioning using Google Pixel 4 and Samsung Galaxy S20 smartphones

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National School of Surveying, University of Otago

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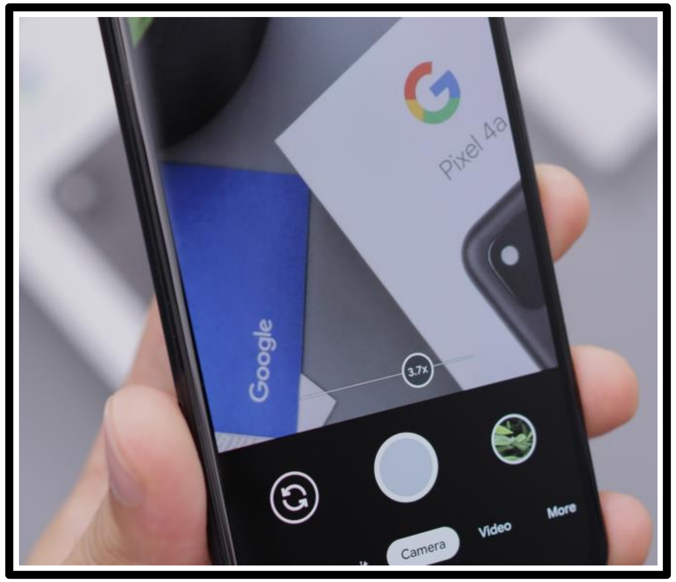
Part 2: RTK Google Pixel5, Pixel6 Pro, Pixel7 Pro and Samsung Galaxy S22.

Dual-frequency and multi-GNSS necessary for successful cm-level smartphone (RTK) positioning

Part 1: RTK Google Pixel 4 and Samsung Galaxy S20, ILS and BIE.

Part 2: RTK Google Pixel5, Pixel6 Pro, Pixel7 Pro and Samsung Galaxy S22.



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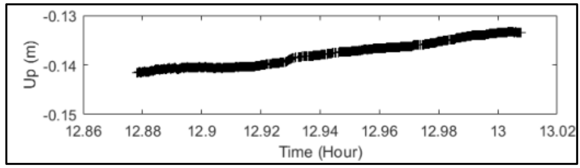
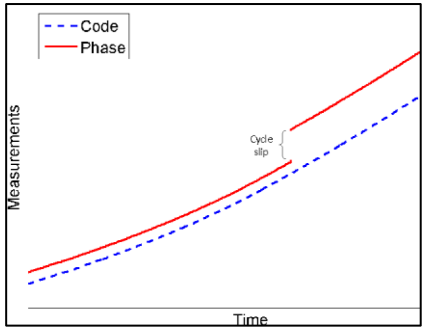
Research motivation

Part 1: RTK Google Pixel 4 and Samsung Galaxy S20, ILS and BIE.

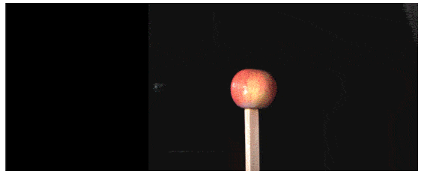
Part 2: RTK Google Pixel5, Pixel6 Pro, Pixel7 Pro and Samsung Galaxy S22.



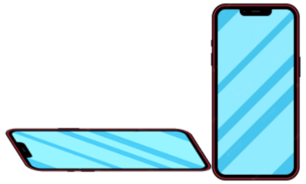
1. All studies on mass-market smartphone data have had issues with integer ambiguity resolution, and some researchers show only small time periods of data with successful cm-level positioning using L1 GPS only (Paziewski et al., 2021; Hesselbarth & Wanninger, 2020).
   - We will show results for up to 12 hours of data using multi-GNSS and dual-frequency data, as collected by two of the newest mass-market smartphones at the time of publication.



2. All studies have used multi-epoch models whereby the unknown parameters are linked in time, making the model stronger.
   - We will use the weaker single-epoch model, but with the added benefit that it becomes insensitive to cycle slips (which are highly present for smartphone antennas).
3. As a consequence of 2 above all studies have required a convergence time for cm-level positioning (typically tens of minutes).
   - We will show instantaneous (single-epoch) cm-level positioning capability for smartphone data.



4. Most (almost all) studies have not considered the orientation of the smartphone antenna.
   - As an unexpected but significant finding of our study, we found that the orientation of the smartphone (antenna) will significantly affect the ambiguity resolution results (more to come…).

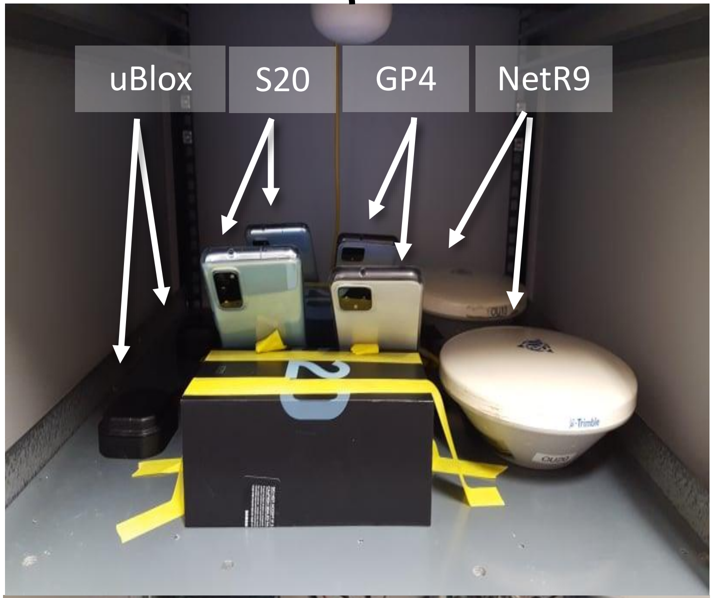


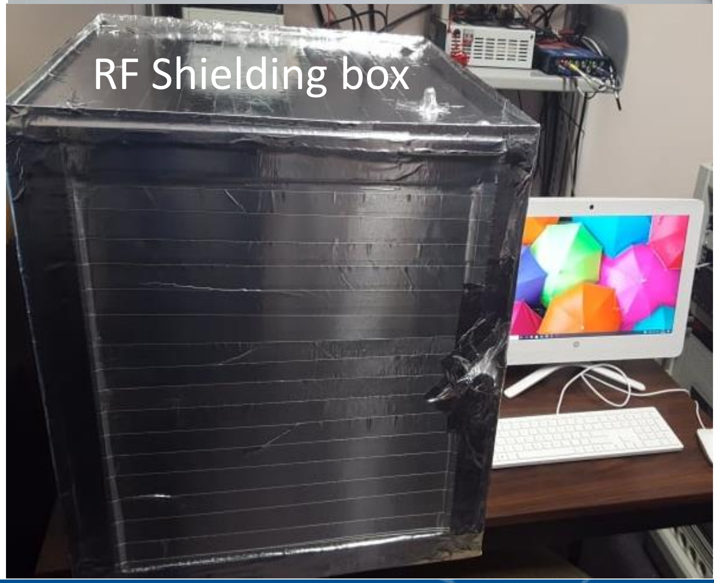
Reference: Yong, C. Z., Odolinski, R., Zaminpardaz, S., Moore, M., Rubinov, E., Er, J., & Denham, M. (2021). Instantaneous, dual-frequency, multi-GNSS precise RTK positioning using google pixel 4 and Samsung Galaxy S20 smartphones for zero and short baselines. Sensors, 21(24), 8318.

Low-cost & smartphone receivers

Samsung Galaxy S20	**G, E, J, C**	**L1+L5, E1 + E5a, L1 + L5, B1**
Google Pixel 4	**G, E, J, C**	**L1+L5, E1 + E5a, L1 + L5, B1**
µBlox F9P	**G, E, J, C**	**L1+L2, E1 + E5b, L1 + L2, B1 + B2**
Trimble NetR9	**G, E, J, C**	**L1+L2, E1 + E5b, L1 + L2, B1 + B2 + B3**

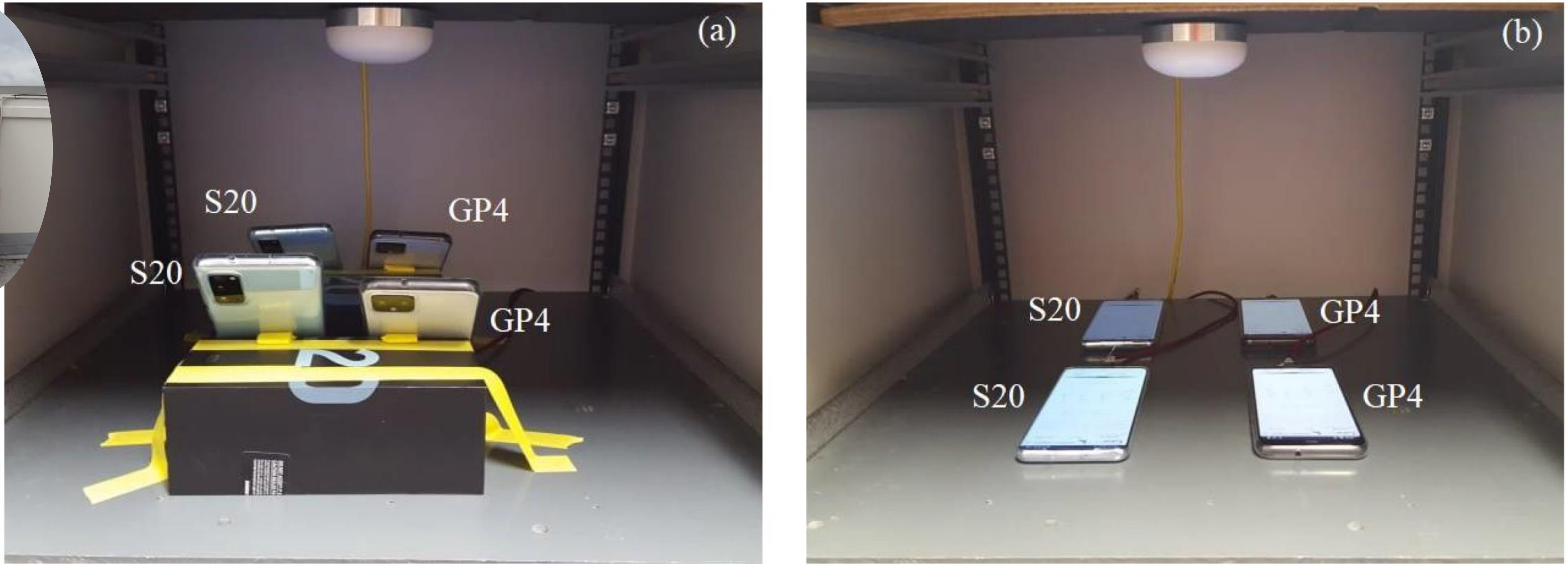
Re-radiating antenna





Reference: Yong, C. Z., Odolinski, R., Zaminpardaz, S., Moore, M., Rubinov, E., Er, J., & Denham, M. (2021). Instantaneous, dual-frequency, multi-GNSS precise RTK positioning using google pixel 4 and Samsung Galaxy S20 smartphones for zero and short baselines. Sensors, 21(24), 8318.

Setup Configurations (1), ZBL external antenna



Zero-baseline (ZBL) setup configurations for smartphones in (**a**) upright and (**b**) lying down positions. The re-radiating antenna receives the GNSS signals through the roof-top antenna

Reference: Yong, C. Z., Odolinski, R., Zaminpardaz, S., Moore, M., Rubinov, E., Er, J., & Denham, M. (2021). Instantaneous, dual-frequency, multi-GNSS precise RTK positioning using google pixel 4 and Samsung Galaxy S20 smartphones for zero and short baselines. Sensors, 21(24), 8318.

Integer Least Squares (ILS)

The single-baseline RTK functional model can be given as,

$$E\left(y\right) = Aa + Bb,\;\; a \in \mathbb{Z}^{n}, b \in \mathbb{R}^{p} \tag{1}$$

where $E\left(.\right)$ is the expectation operator, y the vector of code and phase observations, a is the vector of unknown integer ambiguities, and b vector of real-valued coordinate components (and for sufficiently long baselines, it also includes ionospheric and tropospheric delays). The design matrices A and B are assumed to be of full rank.

Reference: Yong, C. Z., Odolinski, R., Zaminpardaz, S., Moore, M., Rubinov, E., Er, J., & Denham, M. (2021). Instantaneous, dual-frequency, multi-GNSS precise RTK positioning using google pixel 4 and Samsung Galaxy S20 smartphones for zero and short baselines. Sensors, 21(24), 8318.

Integer Least Squares (ILS)

Part 1: RTK Google Pixel 4 and Samsung Galaxy S20, ILS and BIE.

Part 2: RTK Google Pixel5, Pixel6 Pro, Pixel7 Pro and Samsung Galaxy S22.

1) Firstly assume $a \in \mathbb{R}^n$ and perform a least-squares adjustment, to obtain the 'float solution', denoted with a 'hat', and its (co)variance matrices,

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix}, \begin{bmatrix} Q_{\hat{a}\hat{a}} & Q_{\hat{a}\hat{b}} \\ Q_{\hat{b}\hat{a}} & Q_{\hat{b}\hat{b}} \end{bmatrix} \quad (2)$$

Float

2) Secondly decorrelate ambiguities through the LAMBDA method $\hat{z} = Z^T \hat{a}$ to obtain an almost diagonal variance matrix $Q_{\hat{z}\hat{z}} = Z^T Q_{\hat{a}\hat{a}} Z$. We then find the **single** integer candidate vector through an integer search that minimizes the weighted squared norm,

$$\underset{z \in \mathbb{Z}^n}{\arg\min} ||\hat{z} - z||^2_{Q_{\hat{z}\hat{z}}} \quad (3)$$

3) where $||.||^2_{Q_{\hat{z}\hat{z}}} = (.)^T Q^{-1}_{\hat{z}\hat{z}} (.)$. Finally we transform $\check{a} = Z^{-T} \check{z}$ and compute the fixed ILS baseline solution $\check{b}$, denoted with 'check', and its variance matrix,

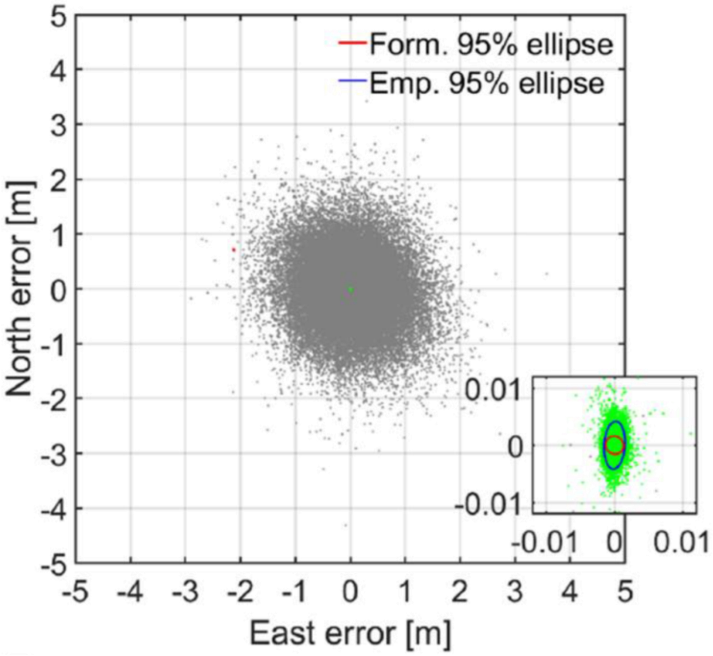
$$\check{b} = \hat{b} - Q_{\hat{b}\hat{a}} Q^{-1}_{\hat{a}\hat{a}} (\hat{a} - \check{a}), \; Q_{\check{b}\check{b}} = Q_{\hat{b}\hat{b}} - Q_{\hat{b}\hat{a}} Q^{-1}_{\hat{a}\hat{a}} Q_{\hat{a}\hat{b}} \quad (4)$$

Fixed ILS

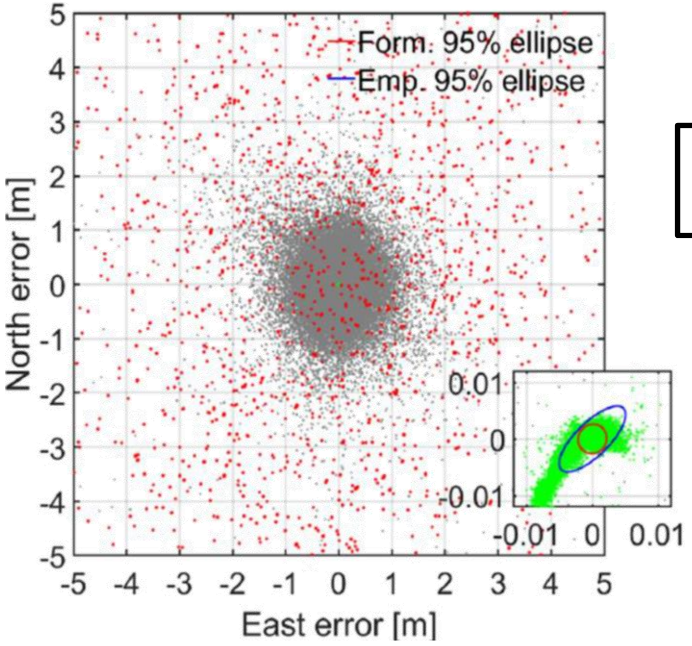
Reference: Yong, C. Z., Odolinski, R., Zaminpardaz, S., Moore, M., Rubinov, E., Er, J., & Denham, M. (2021). Instantaneous, dual-frequency, multi-GNSS precise RTK positioning using google pixel 4 and Samsung Galaxy S20 smartphones for zero and short baselines. Sensors, 21(24), 8318.

ZBL – Lying down position

GP4



S20



$\hat{b} =$ Float solution, i.e. $a \in \mathbb{R}^n$

$\check{b} = \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}\hat{a}}^{-1} (\hat{a} - \check{a})$ Fixed solution

$\check{b}$ when a incorrectly fixed.

GP4 **ILS SR: 99.9%**
(0.1% incorrectly fixed)

Mean ± STDs m (correctly fixed):

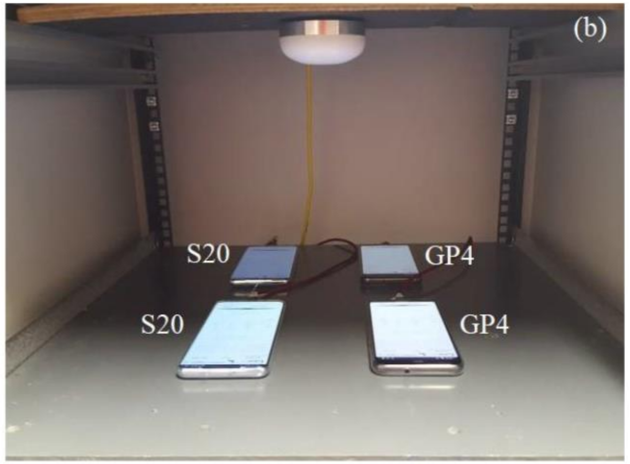
E 0.000 ± 0.001 m
N 0.000 ± 0.002 m
U -0.008 ± 0.003 m

ILS (Integer Least Squares) Success Rate (SR) is the probability of correct integer estimation

S20 **ILS SR: 79.4%**
(20.6% incorrectly fixed)

Mean ± STDs m (correctly fixed):

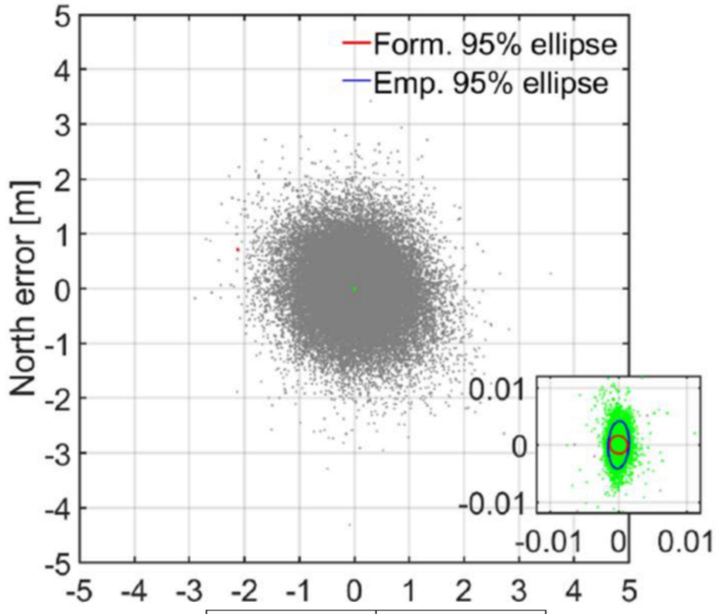
E 0.000 ± 0.002 m
N 0.001 ± 0.002 m
U -0.004 ± 0.006 m

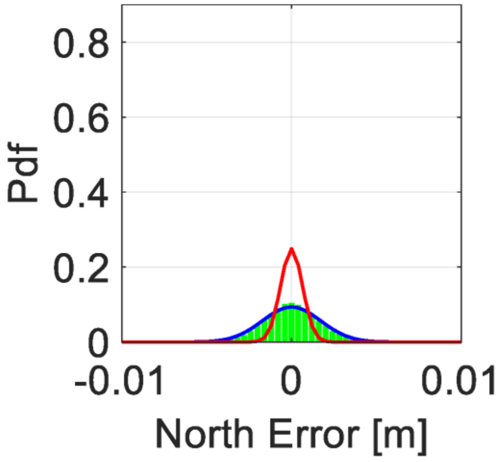


Reference: Yong, C. Z., Odolinski, R., Zaminpardaz, S., Moore, M., Rubinov, E., Er, J., & Denham, M. (2021). Instantaneous, dual-frequency, multi-GNSS precise RTK positioning using google pixel 4 and Samsung Galaxy S20 smartphones for zero and short baselines. Sensors, 21(24), 8318.

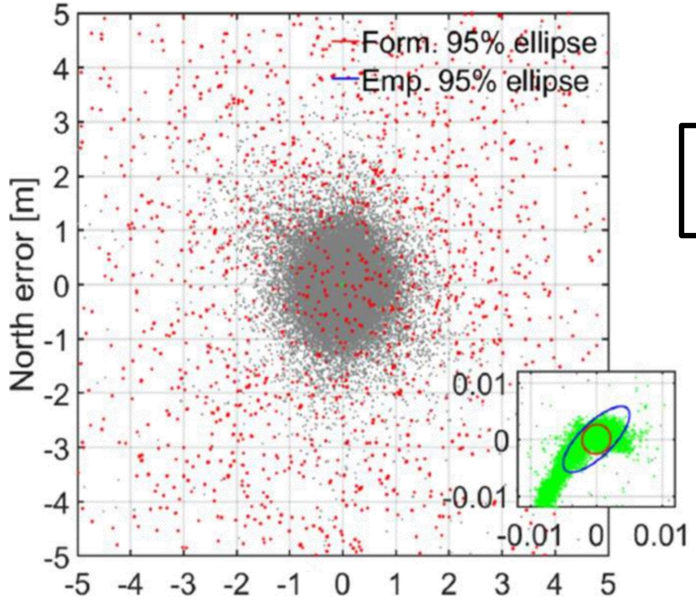
ZBL – Lying down position 

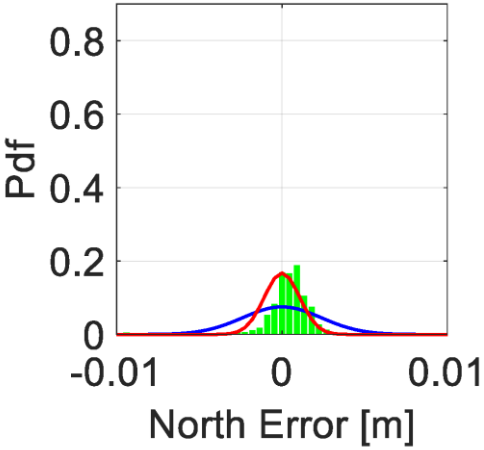
GP4





S20





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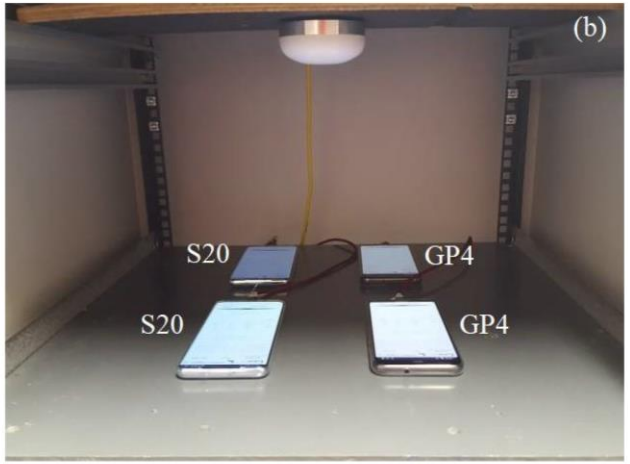

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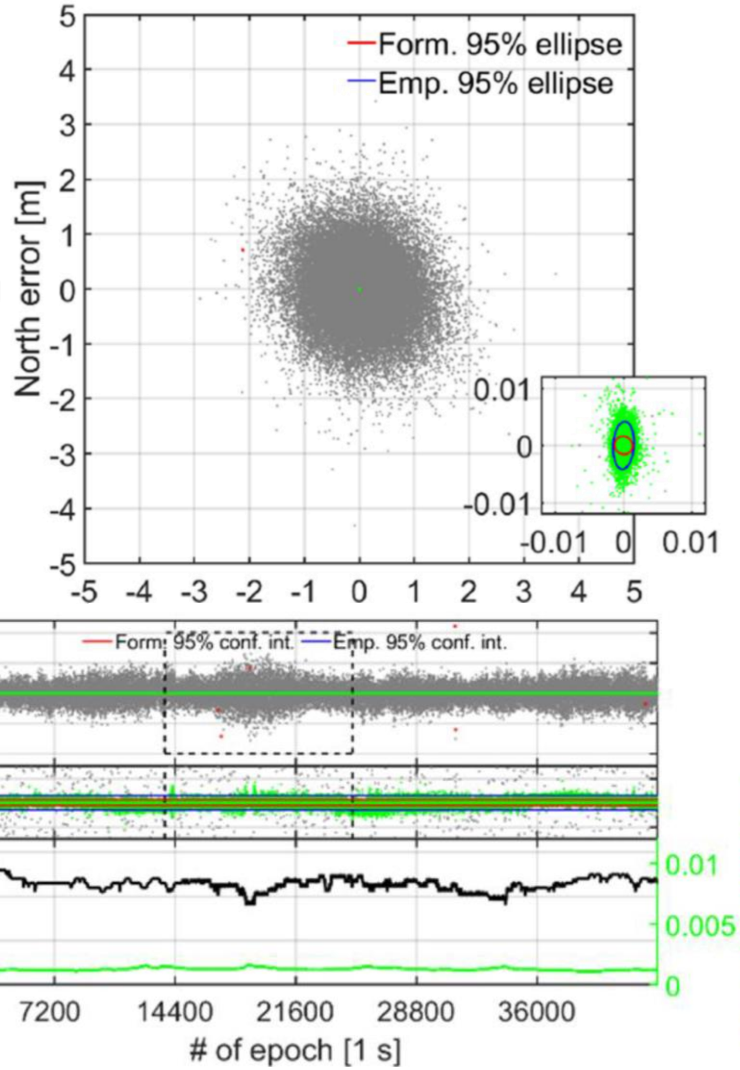
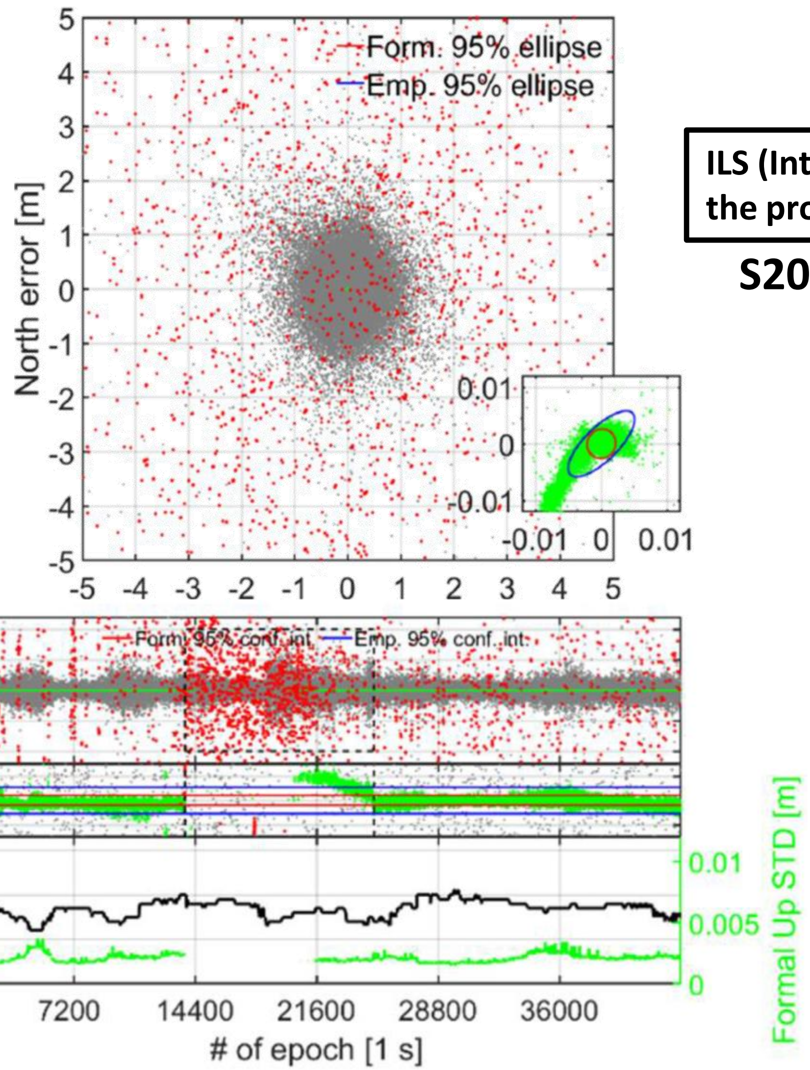
ZBL – Lying down position

GP4



S20



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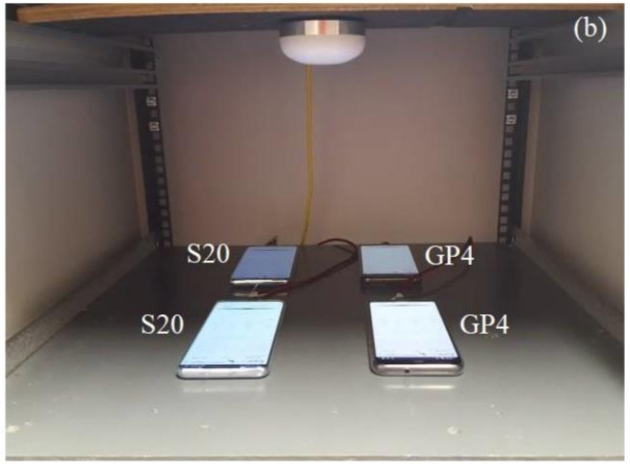

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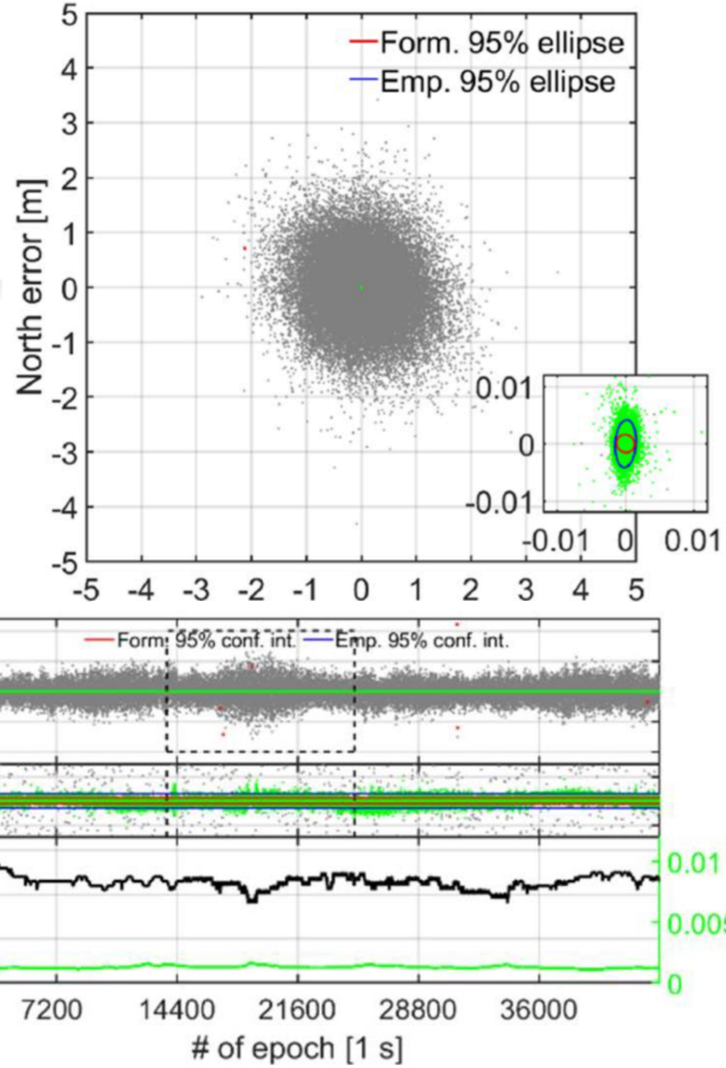
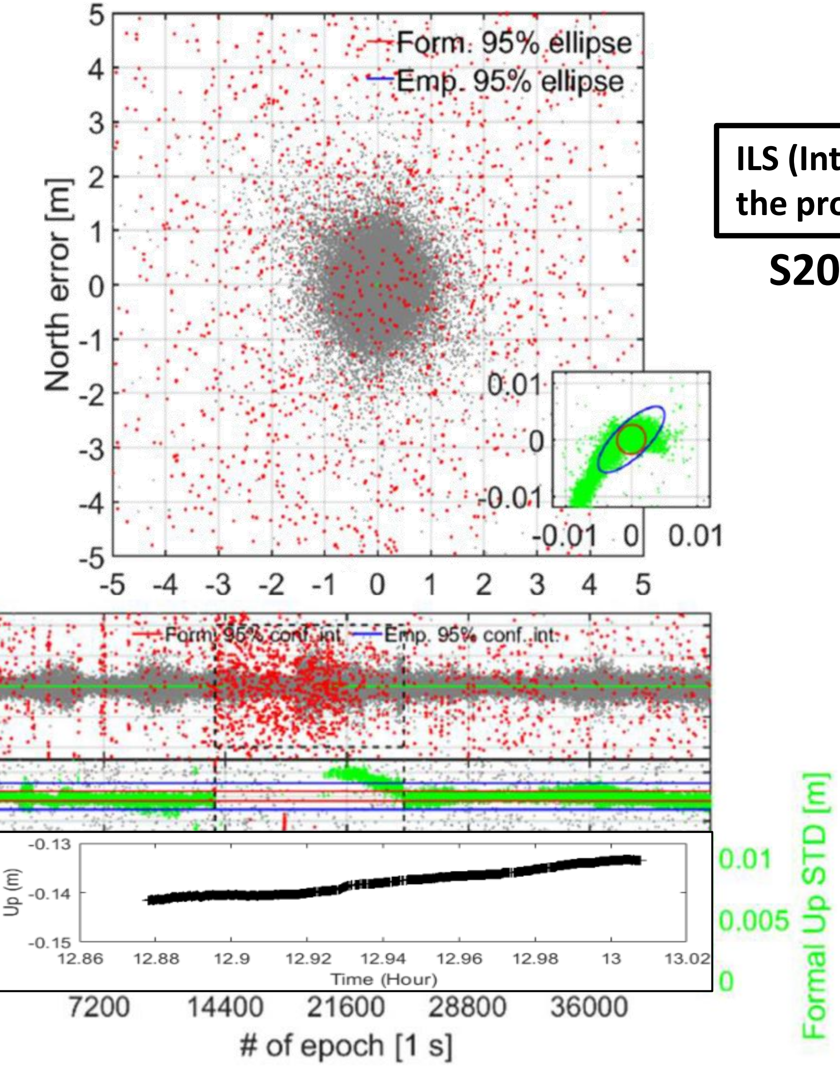
ZBL – Lying down position

GP4



S20



GP4 **ILS SR: 99.9%**

(0.1% incorrectly fixed)

Mean ± STDs m (correctly fixed):

E 0.000 ± 0.001 m

N 0.000 ± 0.002 m

U -0.008 ± 0.003 m

ILS (Integer Least Squares) Success Rate (SR) is the probability of correct integer estimation

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(20.6% incorrectly fixed)

Mean ± STDs m (correctly fixed):

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N 0.001 ± 0.002 m

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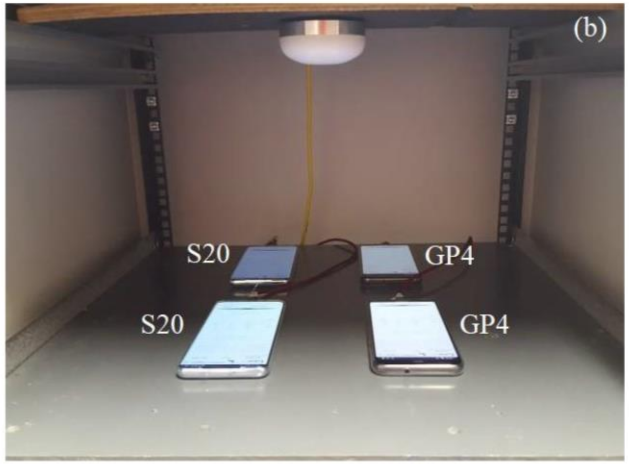

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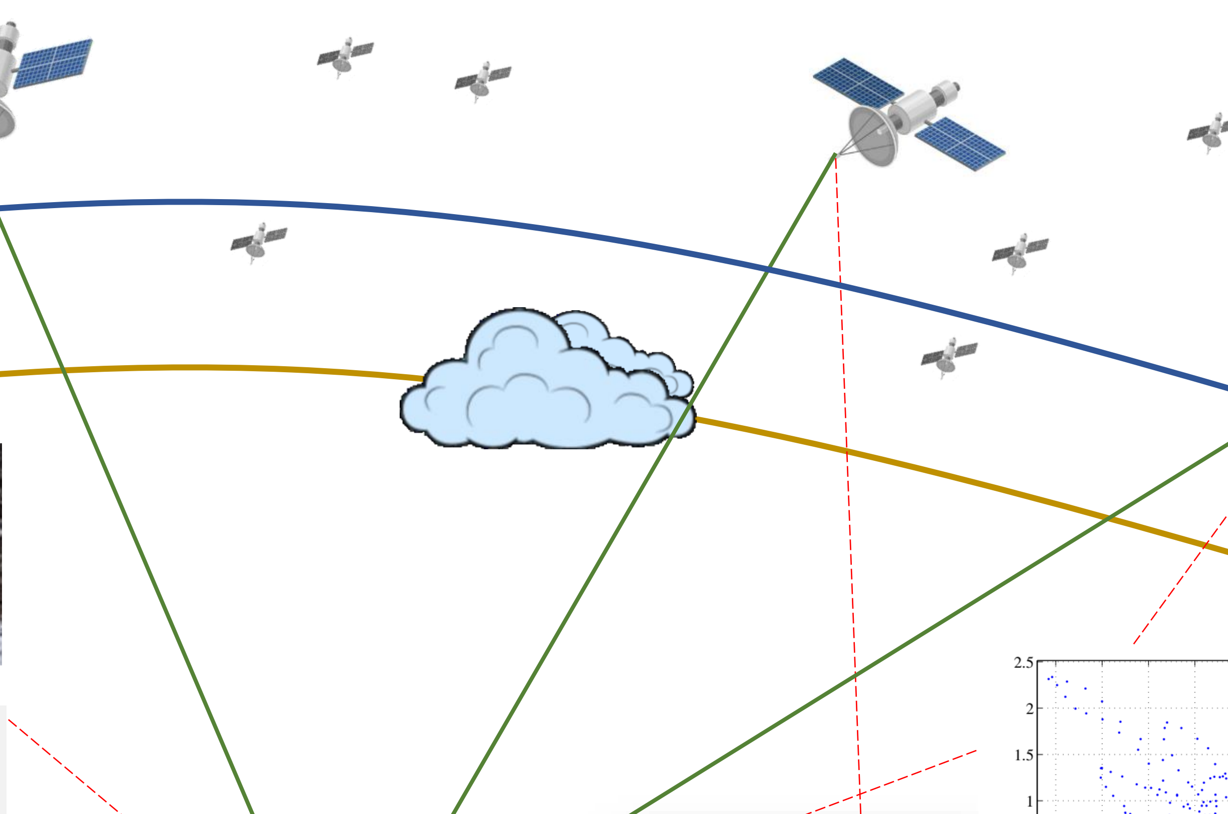
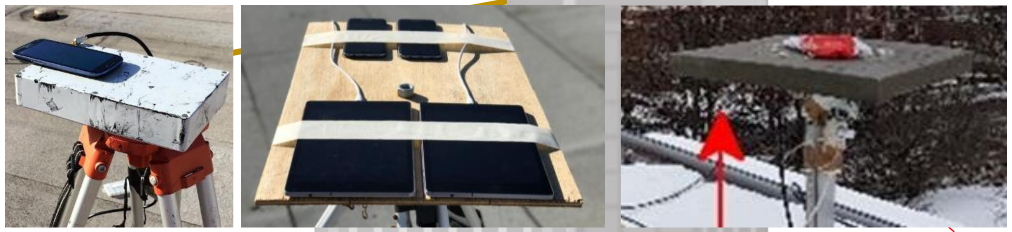
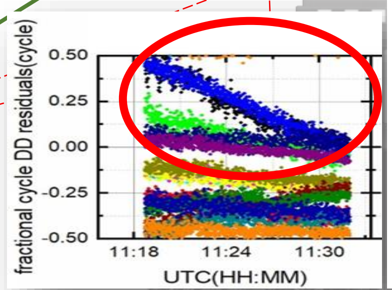
Breakthrough

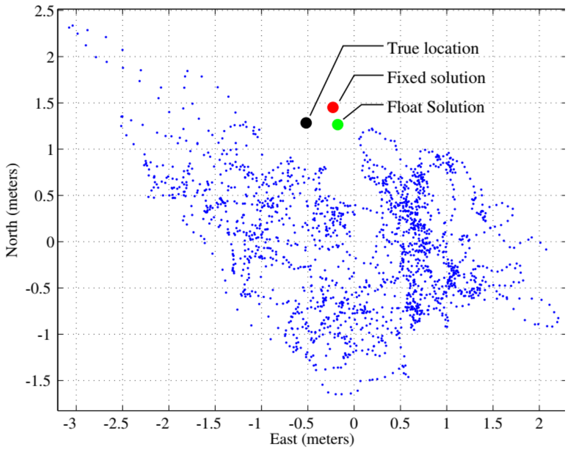




Pesyna et al. (2014)
Humphreys et al. (2016)
Riley et al. (2017)
Håkansson (2019)
Geng&Li (2019)
Wu et al. (2019)
Darugna et al. (2019)
Gao et al. (2021)

Lying down position!

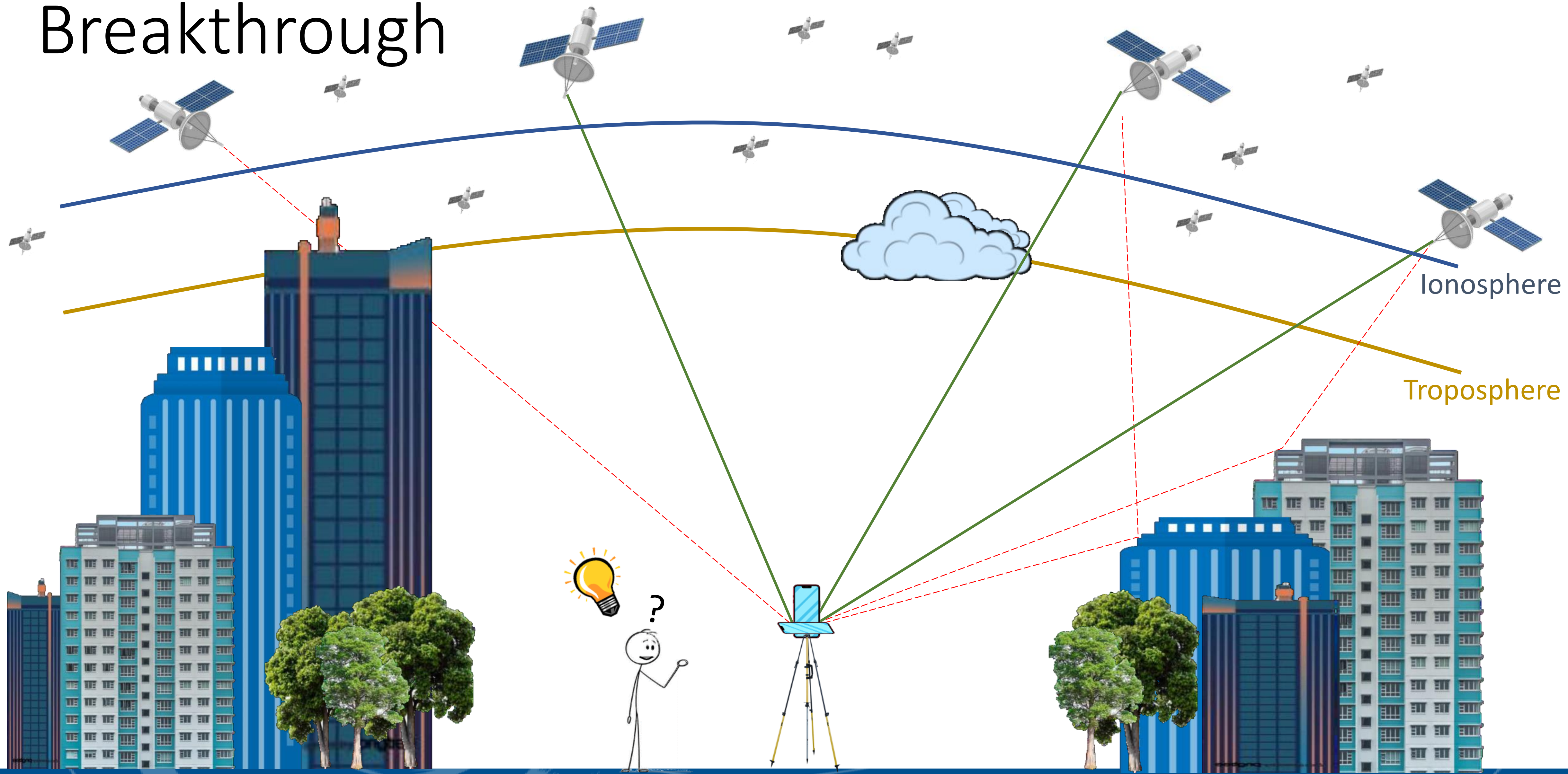




Linear trend and non-zero-mean double-differenced (DD) phase residuals

Reference: Yong, C. Z., Odolinski, R., Zaminpardaz, S., Moore, M., Rubinov, E., Er, J., & Denham, M. (2021). … precise RTK positioning using google pixel 4 and Samsung Galaxy S20 smartphones for zero and short baselines. Sensors, 21(24), 8318.

Breakthrough



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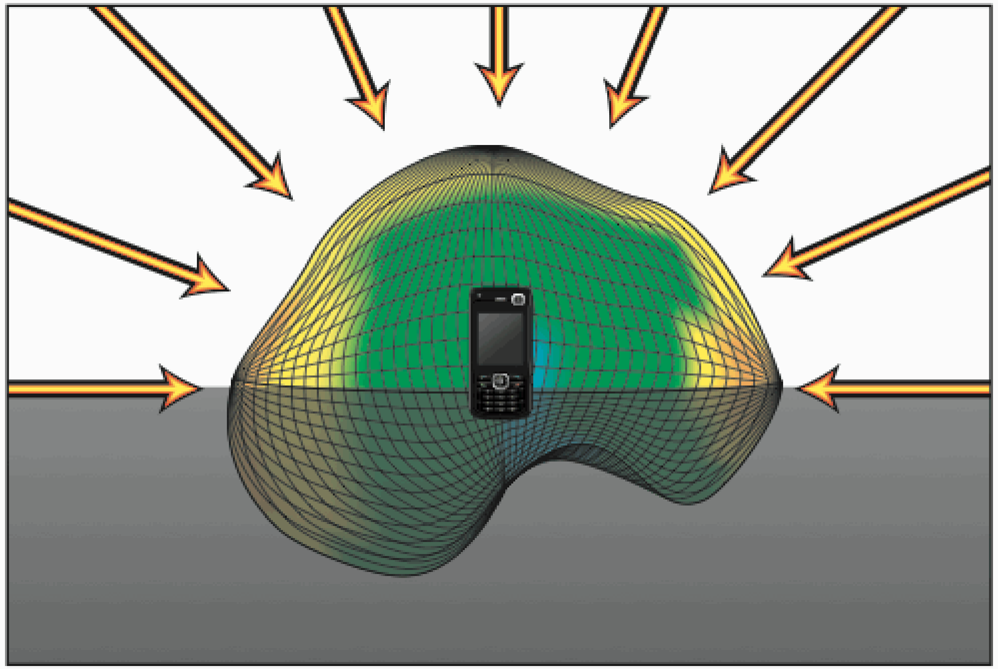
Antenna design

Part 1: RTK Google Pixel 4 and Samsung Galaxy S20, ILS and BIE.

Part 2: RTK Google Pixel5, Pixel6 Pro, Pixel7 Pro and Samsung Galaxy S22.



GNSS antennas in smartphones are generally constructed for the phone to be oriented upright (portrait mode)

Reference: https://www.antenna-theory.com/design/gps.php, viewed 09/11/2023

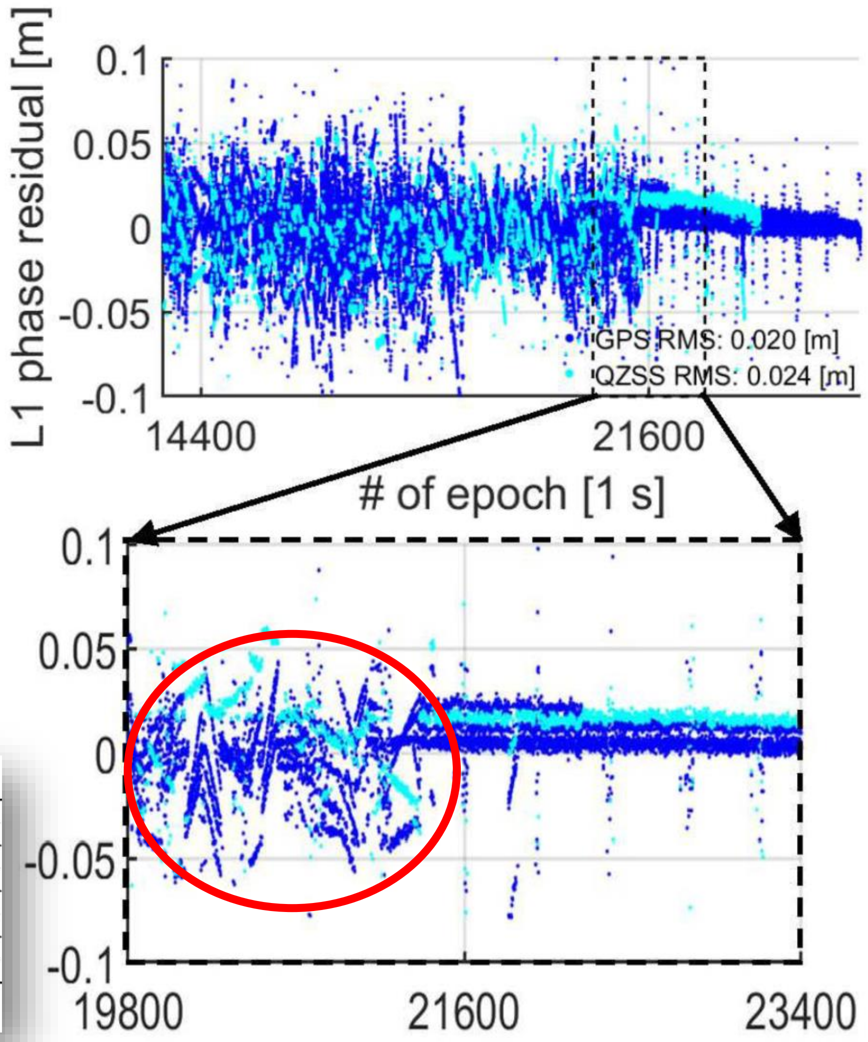
Reference: Yong, C. Z., Odolinski, R., Zaminpardaz, S., Moore, M., Rubinov, E., Er, J., & Denham, M. (2021). Instantaneous, dual-frequency, multi-GNSS precise RTK positioning using google pixel 4 and Samsung Galaxy S20 smartphones for zero and short baselines. Sensors, 21(24), 8318.

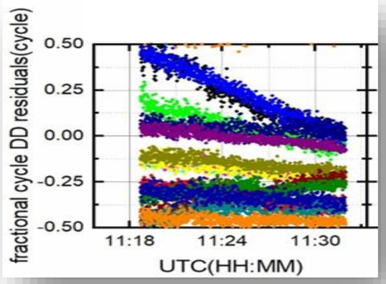
Phase Residual (Samsung S20)

Part 1: RTK Google Pixel 4 and Samsung Galaxy S20, ILS and BIE.

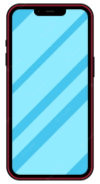
Part 2: RTK Google Pixel5, Pixel6 Pro, Pixel7 Pro and Samsung Galaxy S22.

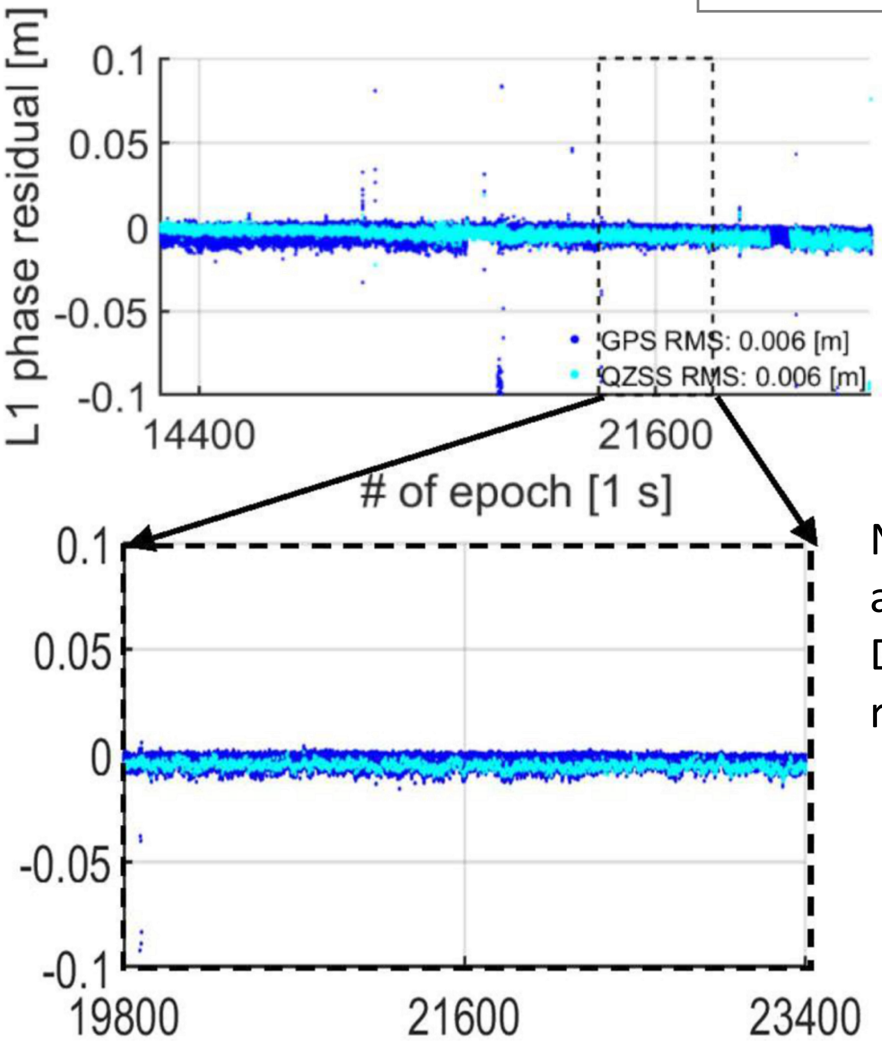
Lying down





Upright



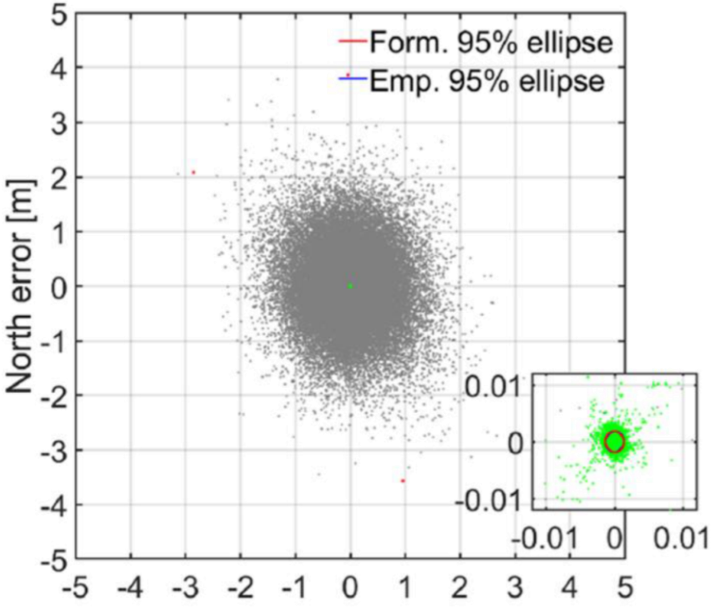


No linear trend and zero-mean DD phase residuals

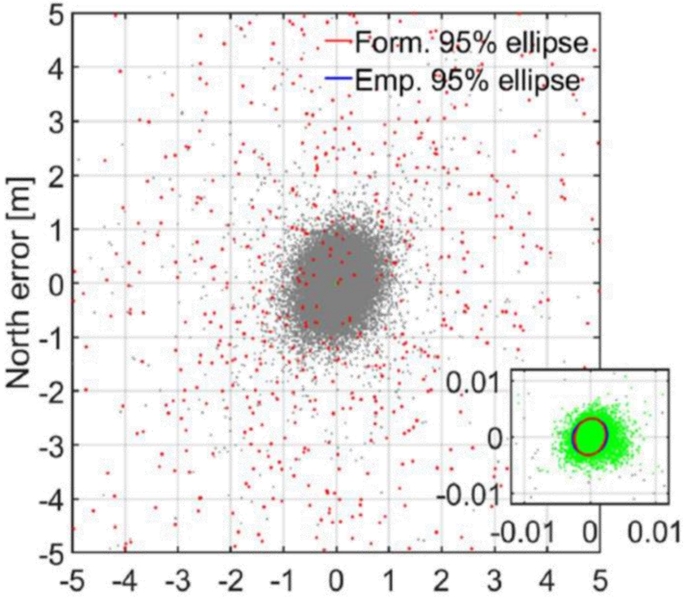
Reference: Yong, C. Z., Odolinski, R., Zaminpardaz, S., Moore, M., Rubinov, E., Er, J., & Denham, M. (2021). Instantaneous, dual-frequency, multi-GNSS precise RTK positioning using google pixel 4 and Samsung Galaxy S20 smartphones for zero and short baselines. Sensors, 21(24), 8318.

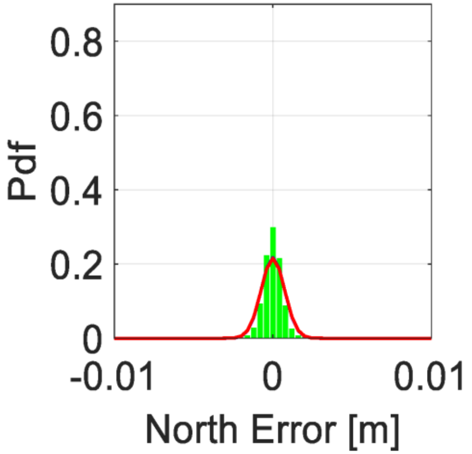
ZBL – Upright position

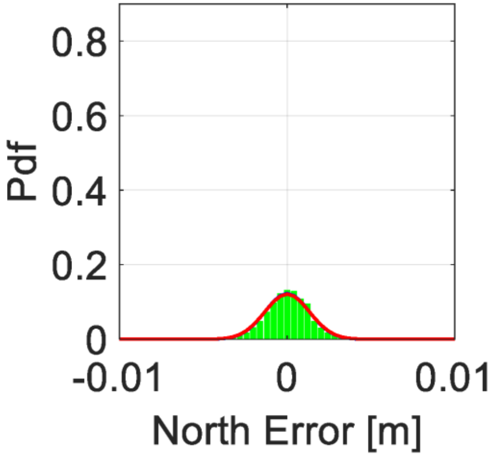

GP4




S20







GP4 **ILS SR: 99.9%**

(0.1% incorrectly fixed)

Mean ± STDs m (correctly fixed):

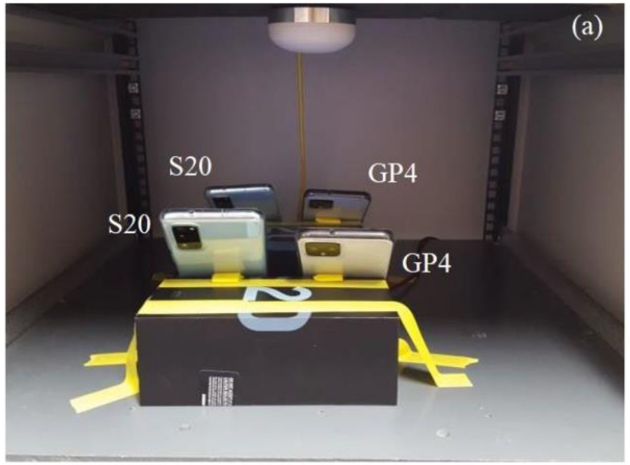
E 0.000 ± 0.001 m
N 0.000 ± 0.001 m
U 0.004 ± 0.003 m

S20 **ILS SR: 97.8% (lying down 79.4%)**

(2.2% incorrectly fixed)

Mean ± STDs m (correctly fixed):

E 0.000 ± 0.001 m
N 0.000 ± 0.001 m
U 0.001 ± 0.003 m



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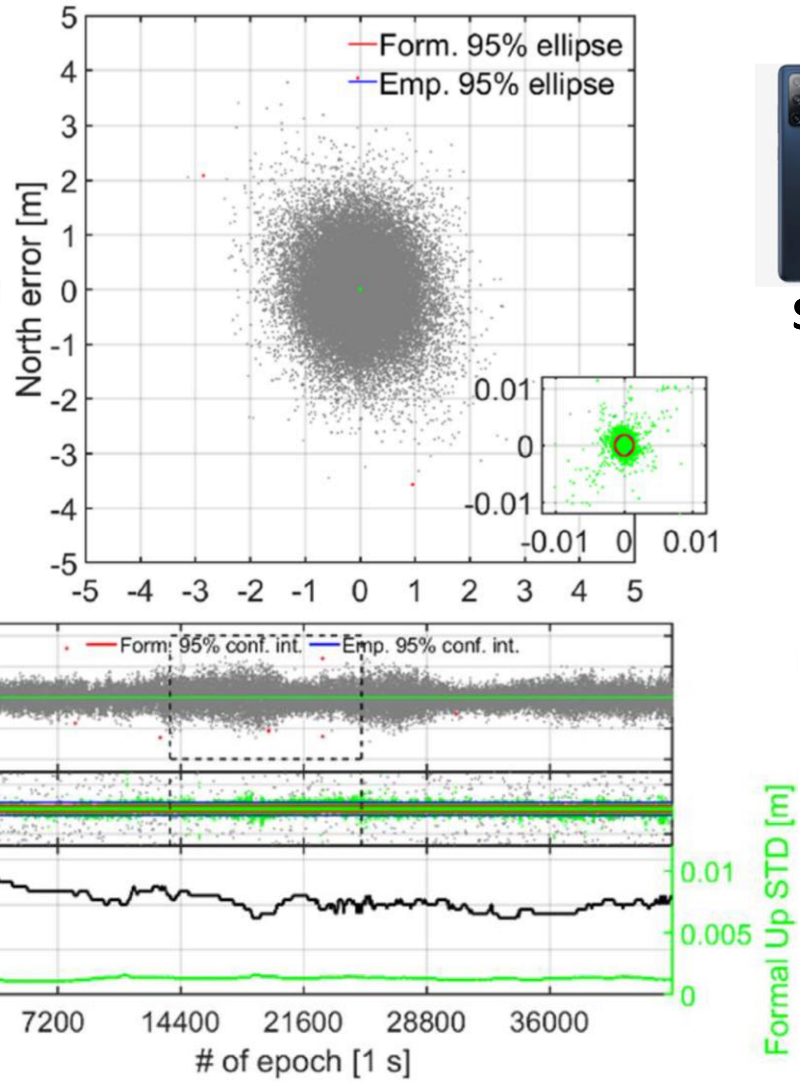
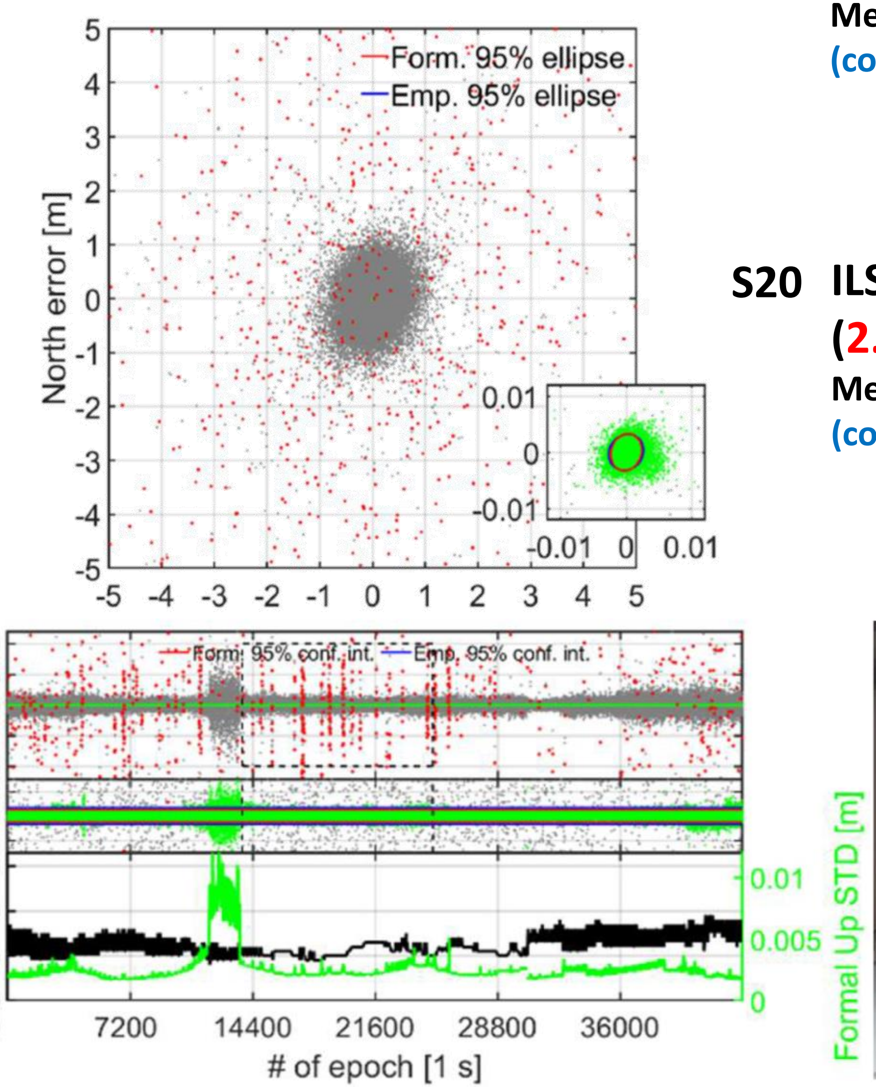
ZBL – Upright position

GP4



S20



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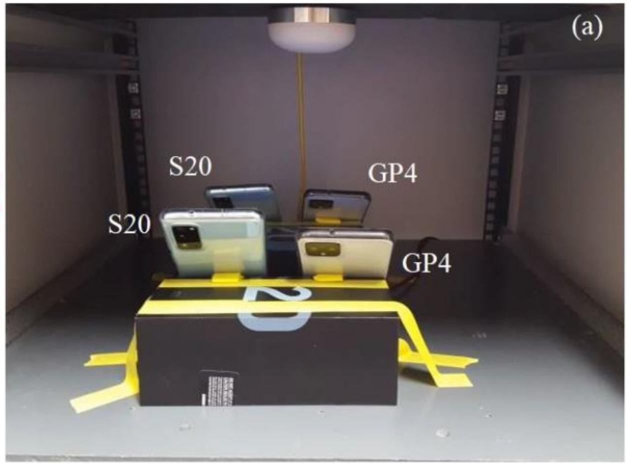
N 0.000 ± 0.001 m
U 0.004 ± 0.003 m

S20 **ILS SR: 97.8% (lying down 79.4%)**

(2.2% incorrectly fixed)

Mean ± STDs m (correctly fixed):

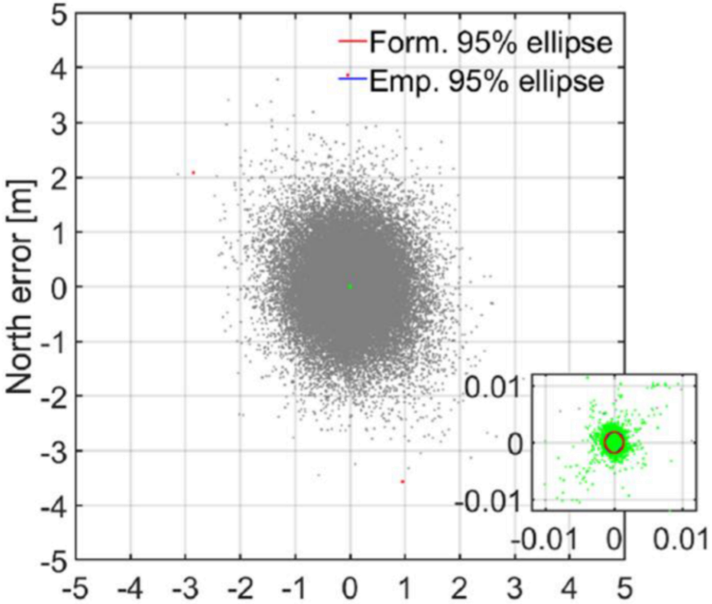
E 0.000 ± 0.001 m
N 0.000 ± 0.001 m
U 0.001 ± 0.003 m

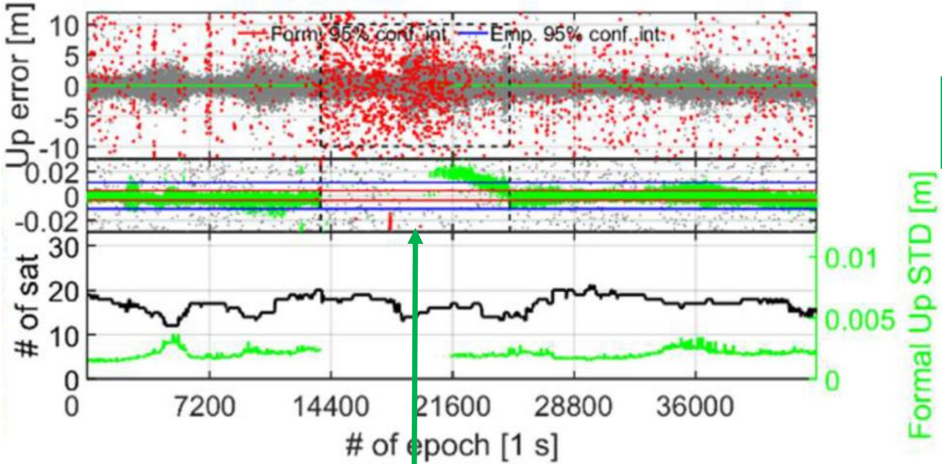


Reference: Yong, C. Z., Odolinski, R., Zaminpardaz, S., Moore, M., Rubinov, E., Er, J., & Denham, M. (2021). Instantaneous, dual-frequency, multi-GNSS precise RTK positioning using google pixel 4 and Samsung Galaxy S20 smartphones for zero and short baselines. Sensors, 21(24), 8318.

ZBL – Upright position

GP4





GP4 **ILS SR: 99.9%**

(0.1% incorrectly fixed)

Mean ± STDs m (correctly fixed):

E 0.000 ± 0.001 m

N 0.000 ± 0.001 m

U 0.004 ± 0.003 m

ILS SR: 97.8% (lying down 79.4%)

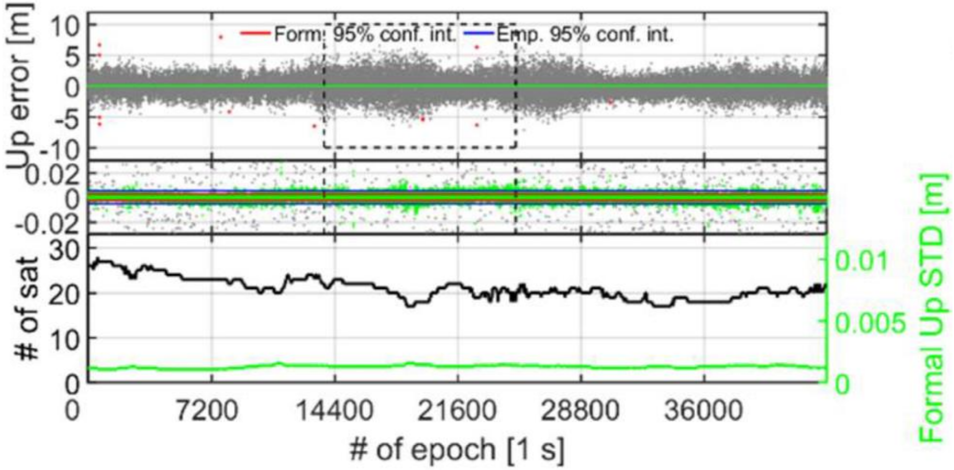
(2.2% incorrectly fixed)

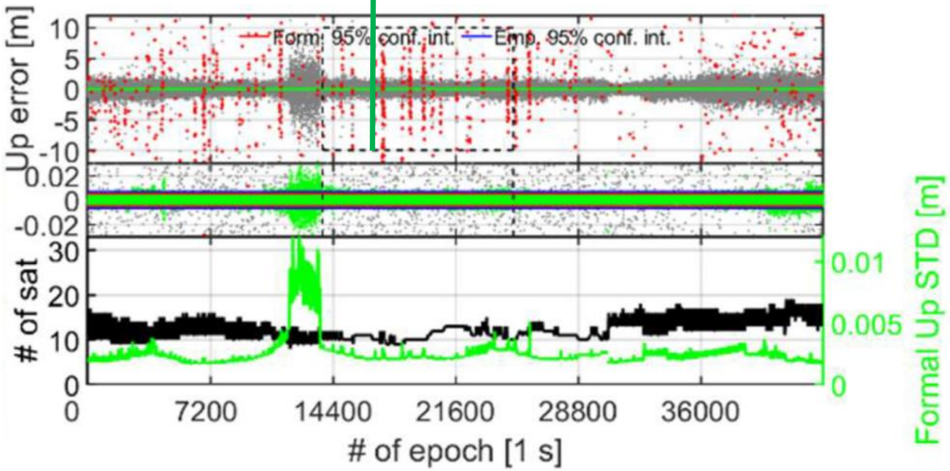
Mean ± STDs m (correctly fixed):

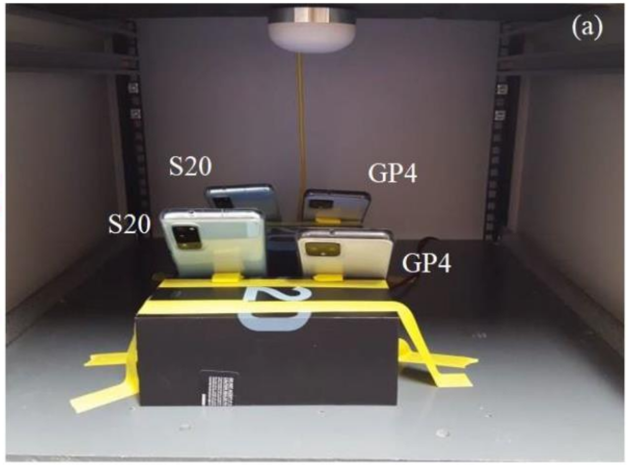
E 0.000 ± 0.001 m

N 0.000 ± 0.001 m

U 0.001 ± 0.003 m







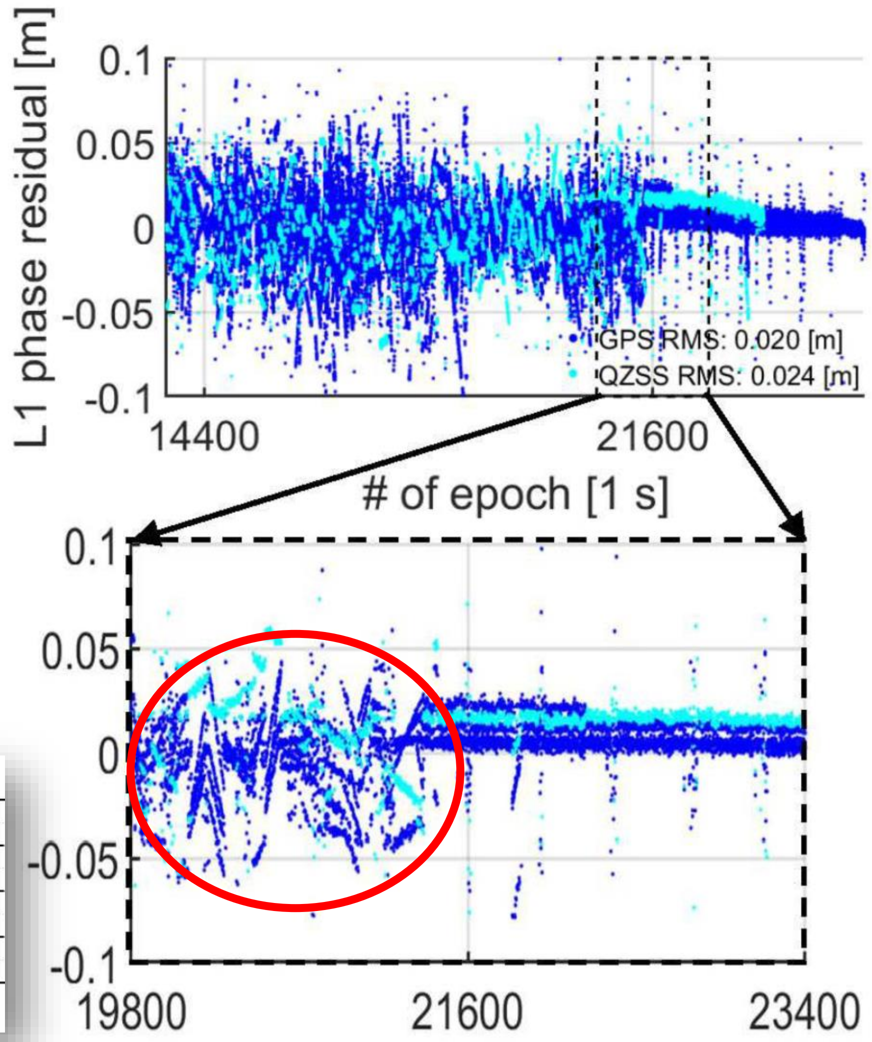
Reference: Yong, C. Z., Odolinski, R., Zaminpardaz, S., Moore, M., Rubinov, E., Er, J., & Denham, M. (2021). Instantaneous, dual-frequency, multi-GNSS precise RTK positioning using google pixel 4 and Samsung Galaxy S20 smartphones for zero and short baselines. Sensors, 21(24), 8318.

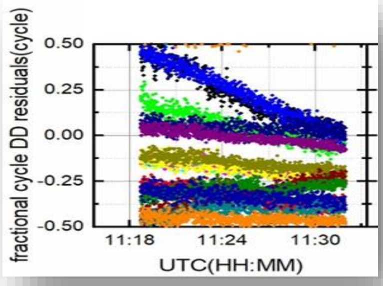
Phase Residual (Samsung S20)

Part 1: RTK Google Pixel 4 and Samsung Galaxy S20, ILS and BIE.

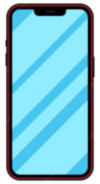
Part 2: RTK Google Pixel5, Pixel6 Pro, Pixel7 Pro and Samsung Galaxy S22.

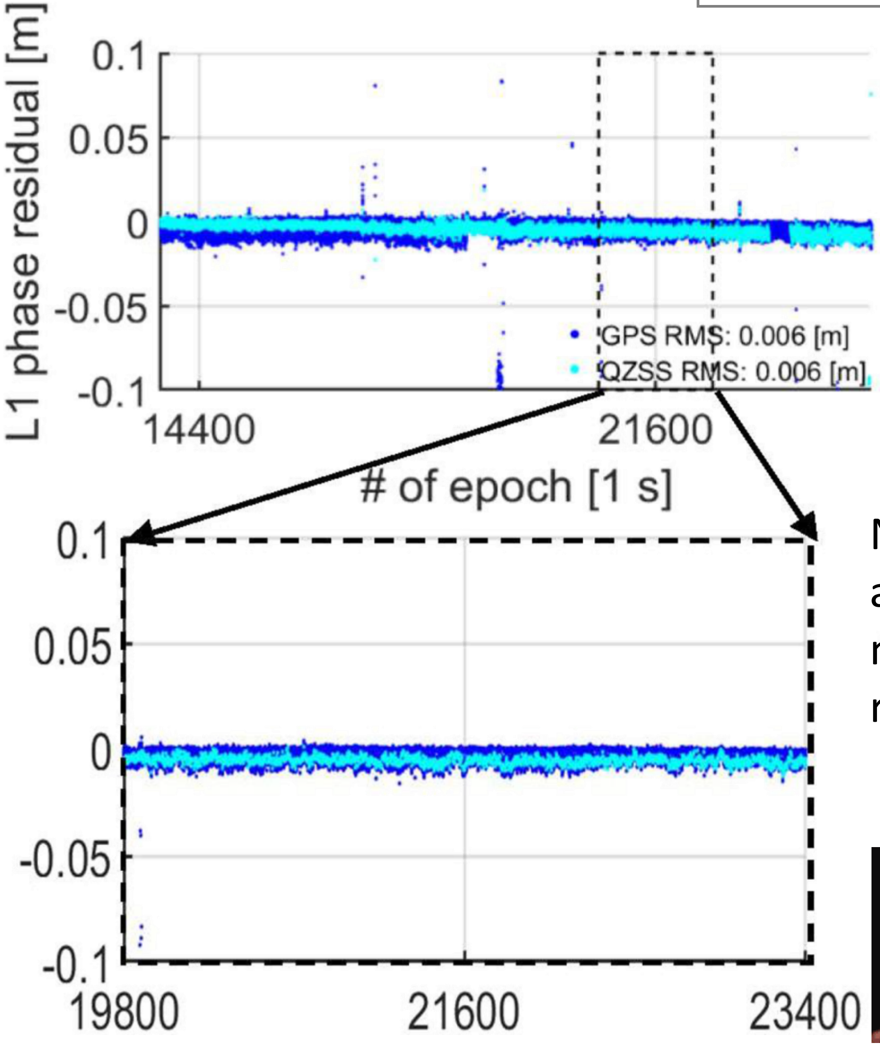
Lying down





Upright





No linear trend and non-zero-mean DD phase residuals

Reference: Yong, C. Z., Odolinski, R., Zaminpardaz, S., Moore, M., Rubinov, E., Er, J., & Denham, M. (2021). Instantaneous, dual-frequency, multi-GNSS precise RTK positioning using google pixel 4 and Samsung Galaxy S20 smartphones for zero and short baselines. Sensors, 21(24), 8318.

Setup Configurations (3), SBL internal antennas

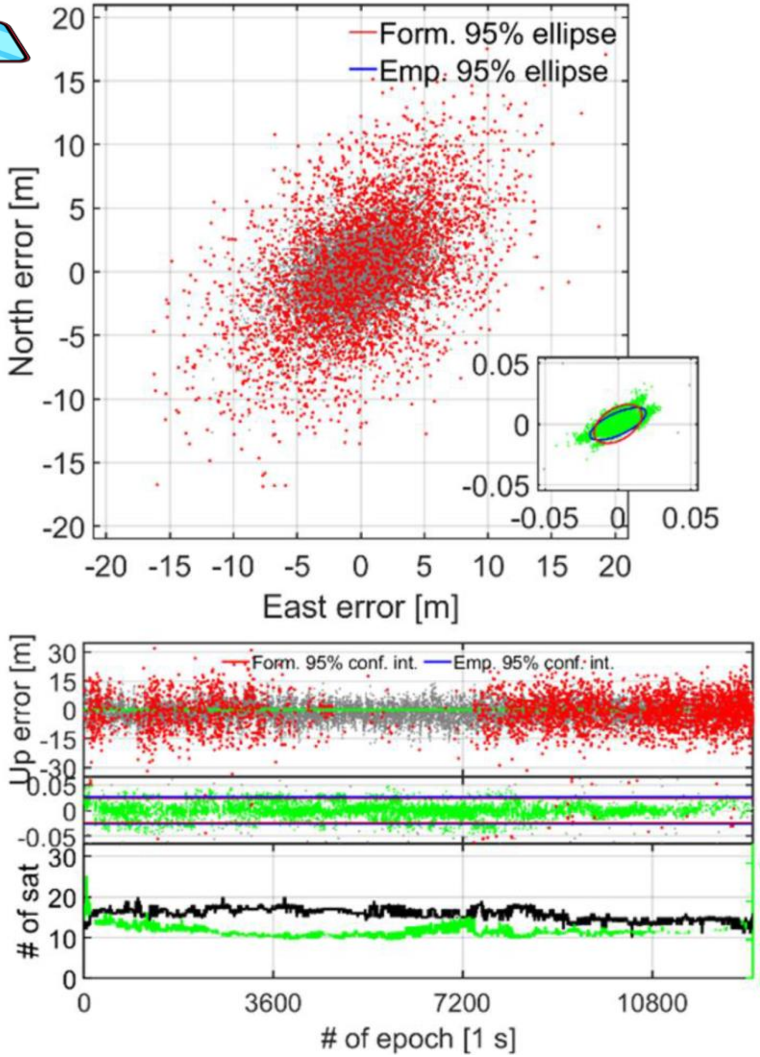
Part 1: RTK Google Pixel 4 and Samsung Galaxy S20, ILS and BIE.

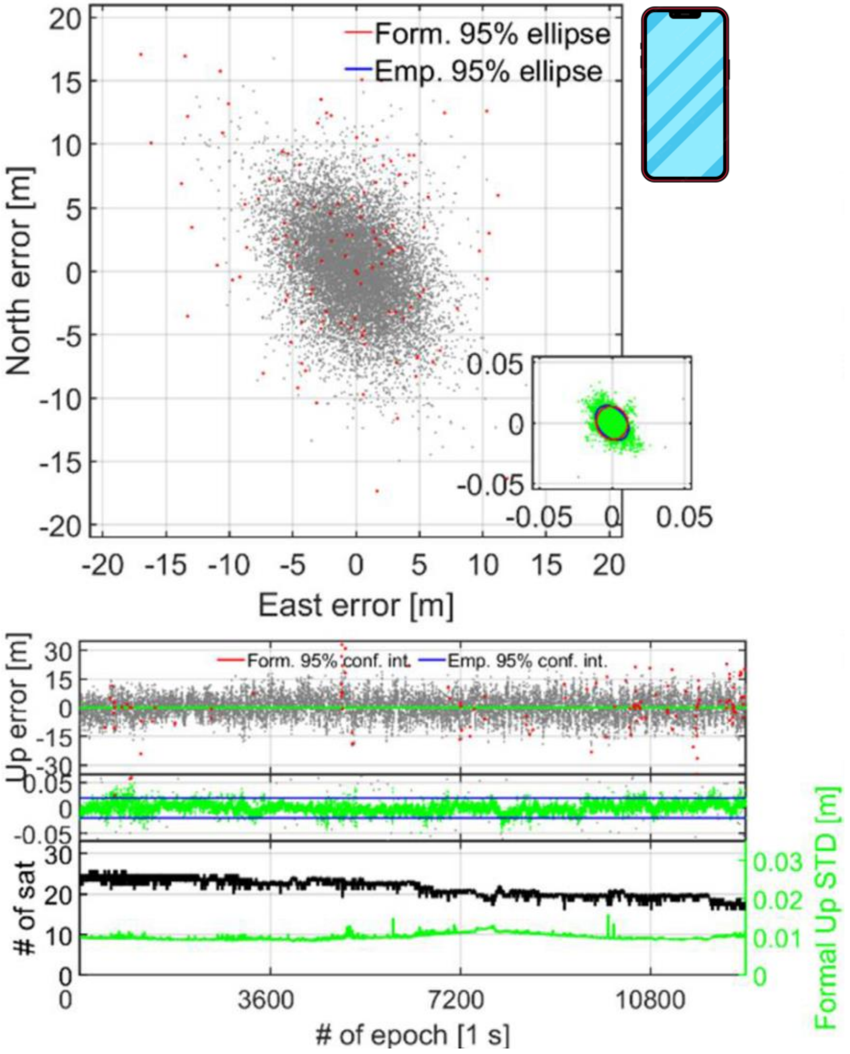
Part 2: RTK Google Pixel5, Pixel6 Pro, Pixel7 Pro and Samsung Galaxy S22.

Short-baseline setup configurations with smartphones internal antennas in upright (**a**,**b**) and lying down (**c**,**d**) positions.

Reference: Yong, C. Z., Odolinski, R., Zaminpardaz, S., Moore, M., Rubinov, E., Er, J., & Denham, M. (2021). Instantaneous, dual-frequency, multi-GNSS precise RTK positioning using google pixel 4 and Samsung Galaxy S20 smartphones for zero and short baselines. Sensors, 21(24), 8318.

Short baseline using internal antennas





GP4

GP4 (internal; lying down)
ILS SR: 63.1%
(36.9% incorrectly fixed)

Mean ± STDs m (correctly fixed):
E -0.002 ± 0.008 m
N -0.000 ± 0.005 m
U -0.002 ± 0.013 m

GP4 (internal; upright)
ILS SR: 98.7%
(1.3% incorrectly fixed)

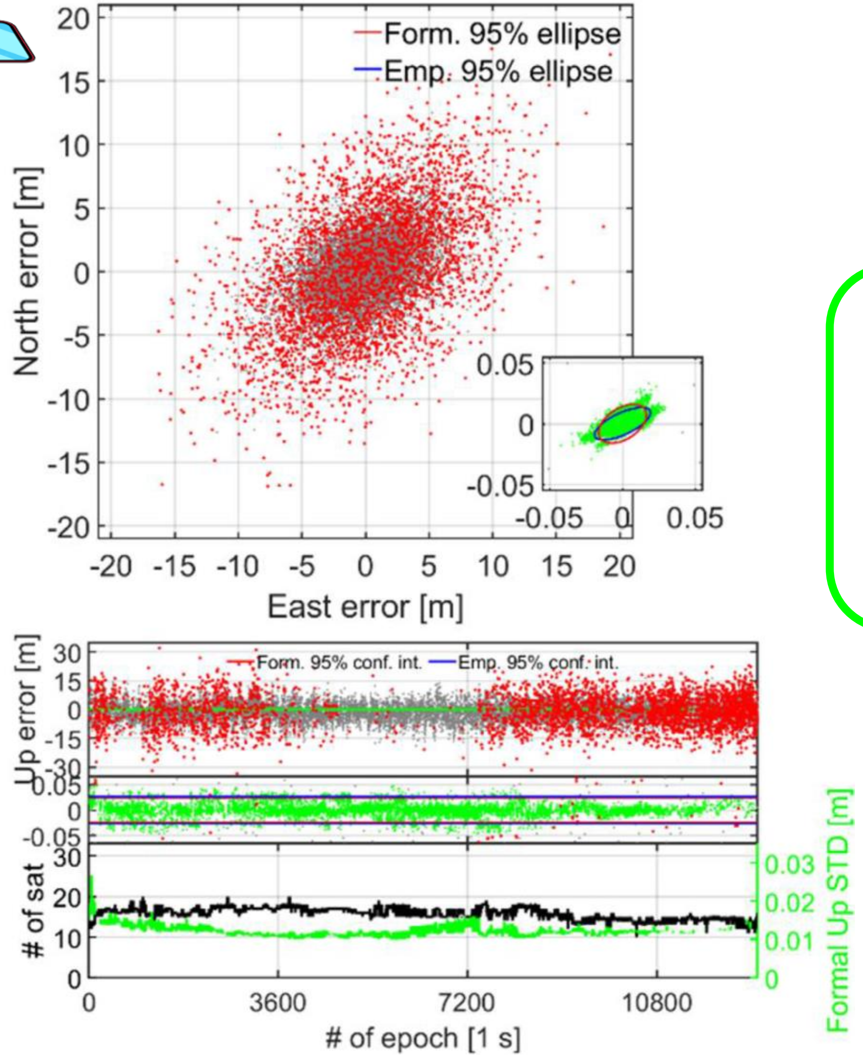
Mean ± STDs m (correctly fixed):
E 0.001 ± 0.005 m
N -0.001 ± 0.006 m
U 0.001 ± 0.010 m

GP4 (external; upright)
ILS SR: 99.9%
(0.1% incorrectly fixed)

Mean ± STDs m (correctly fixed):
E -0.001 ± 0.002 m
N -0.000 ± 0.002 m
U 0.000 ± 0.005 m

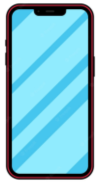
Reference: Yong, C. Z., Odolinski, R., Zaminpardaz, S., Moore, M., Rubinov, E., Er, J., & Denham, M. (2021). Instantaneous, dual-frequency, multi-GNSS precise RTK positioning using google pixel 4 and Samsung Galaxy S20 smartphones for zero and short baselines. Sensors, 21(24), 8318.

Short baseline using internal antennas



GP4

This is groundbreaking stuff mate!



GP4 (internal; lying down)
ILS SR: 63.1%
(36.9% incorrectly fixed)
Mean ± STDs m (correctly fixed): E -0.002 ± 0.008 m, N -0.000 ± 0.005 m, U -0.002 ± 0.013 m

GP4 (internal; upright)
ILS SR: 98.7%
(1.3% incorrectly fixed)
Mean ± STDs m (correctly fixed): E 0.001 ± 0.005 m, N -0.001 ± 0.006 m, U 0.001 ± 0.010 m

GP4 (external; upright)
ILS SR: 99.9%
(0.1% incorrectly fixed)
Mean ± STDs m (correctly fixed): E -0.001 ± 0.002 m, N -0.000 ± 0.002 m, U 0.000 ± 0.005 m

Reference: Yong, C. Z., Odolinski, R., Zaminpardaz, S., Moore, M., Rubinov, E., Er, J., & Denham, M. (2021). Instantaneous, dual-frequency, multi-GNSS precise RTK positioning using google pixel 4 and Samsung Galaxy S20 smartphones for zero and short baselines. Sensors, 21(24), 8318.

Best Integer Equivariant (BIE) vs ILS

1) Firstly assume $a \in \mathbb{R}^n$ and perform a least-squares adjustment, to obtain the 'float solution', denoted with a 'hat', and its (co)variance matrices,

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix}, \begin{bmatrix} Q_{\hat{a}\hat{a}} & Q_{\hat{a}\hat{b}} \\ Q_{\hat{b}\hat{a}} & Q_{\hat{b}\hat{b}} \end{bmatrix} \tag{2}$$

Float

2) Secondly decorrelate ambiguities through the LAMBDA method $\hat{z} = Z^T\hat{a}$ to obtain an almost diagonal variance matrix $Q_{\hat{z}\hat{z}} = Z^T Q_{\hat{a}\hat{a}} Z$. We then find the **single** integer candidate vector through an integer search that minimizes the weighted squared norm,

$$\underset{z \in \mathbb{Z}^n}{\arg\min} ||\hat{z} - z||^2_{Q_{\hat{z}\hat{z}}} \tag{3}$$

3) where $||.||^2_{Q_{\hat{z}\hat{z}}} = (.)^T Q^{-1}_{\hat{z}\hat{z}} (.)$. Finally we transform $\check{a} = Z^{-T}\check{z}$ and compute the fixed ILS baseline solution $\check{b}$, denoted with 'check', and its variance matrix,

$$\check{b} = \hat{b} - Q_{\hat{b}\hat{a}} Q^{-1}_{\hat{a}\hat{a}} (\hat{a} - \check{a}), \; Q_{\check{b}\check{b}} = Q_{\hat{b}\hat{b}} - Q_{\hat{b}\hat{a}} Q^{-1}_{\hat{a}\hat{a}} Q_{\hat{a}\hat{b}} \tag{4}$$

Fixed ILS

Reference: Odolinski, R., & Teunissen, P. J. G. (2020). Best integer equivariant estimation: Performance analysis using real data collected by low-cost, single- and dual- frequency, multi-GNSS receivers for short- to long-baseline RTK positioning. Journal of Geodesy, 94, 91.

Best Integer Equivariant (BIE) vs ILS

1) Firstly assume $a \in \mathbb{R}^n$ and perform a least-squares adjustment, to obtain the 'float solution', denoted with a 'hat', and its (co)variance matrices,

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix}, \begin{bmatrix} Q_{\hat{a}\hat{a}} & Q_{\hat{a}\hat{b}} \\ Q_{\hat{b}\hat{a}} & Q_{\hat{b}\hat{b}} \end{bmatrix} \quad (2)$$

Float

2) Secondly decorrelate ambiguities through the LAMBDA method $\hat{z} = Z^T \hat{a}$ to obtain an almost diagonal variance matrix $Q_{\hat{z}\hat{z}} = Z^T Q_{\hat{a}\hat{a}} Z$.

The **BIE estimator** on the other hand is defined by an infinite weighted sum over the whole space of integers as follows,

$$\bar{a} = \sum_{z \in \mathbb{Z}^n} z \frac{\exp\left(-\frac{1}{2}\|\hat{a} - z\|^2_{Q_{\hat{a}\hat{a}}}\right)}{\sum_{z \in \mathbb{Z}^n} \exp\left(-\frac{1}{2}\|\hat{a} - z\|^2_{Q_{\hat{a}\hat{a}}}\right)} \quad (5)$$

3) The fixed BIE baseline solution $\bar{b}$ can then be derived, denoted with 'overline',

$$\bar{b} = \hat{b} - Q_{\hat{b}\hat{a}} Q^{-1}_{\hat{a}\hat{a}} (\hat{a} - \bar{a}) \quad (7)$$

Fixed BIE

Part 1: RTK Google Pixel 4 and Samsung Galaxy S20, ILS and BIE.

Part 2: RTK Google Pixel5, Pixel6 Pro, Pixel7 Pro and Samsung Galaxy S22.

Reference: Odolinski, R., & Teunissen, P. J. G. (2020). Best integer equivariant estimation: Performance analysis using real data collected by low-cost, single- and dual- frequency, multi-GNSS receivers for short- to long-baseline RTK positioning. Journal of Geodesy, 94, 91.

Best Integer Equivariant (BIE) vs ILS

Part 1: RTK Google Pixel 4 and Samsung Galaxy S20, ILS and BIE.

Part 2: RTK Google Pixel5, Pixel6 Pro, Pixel7 Pro and Samsung Galaxy S22.

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix}, \begin{bmatrix} Q_{\hat{a}\hat{a}} & Q_{\hat{a}\hat{b}} \\ Q_{\hat{b}\hat{a}} & Q_{\hat{b}\hat{b}} \end{bmatrix} \quad (2)$$

Float

$$\arg\min_{z \in \mathbb{Z}^n} ||\hat{z} - z||^2_{Q_{\hat{z}\hat{z}}} \quad (3)$$

$$\check{b} = \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}\hat{a}}^{-1} (\hat{a} - \check{a}), \; Q_{\check{b}\check{b}} = Q_{\hat{b}\hat{b}} - Q_{\hat{b}\hat{a}} Q_{\hat{a}\hat{a}}^{-1} Q_{\hat{a}\hat{b}} \quad (4)$$

Fixed ILS

$$\bar{a} = \sum_{z \in \mathbb{Z}^n} z \frac{\exp\left(-\frac{1}{2}||\hat{a} - z||^2_{Q_{\hat{a}\hat{a}}}\right)}{\sum_{z \in \mathbb{Z}^n} \exp\left(-\frac{1}{2}||\hat{a} - z||^2_{Q_{\hat{a}\hat{a}}}\right)} \quad (5)$$

$$||\hat{a} - z||^2_{Q_{\hat{a}\hat{a}}} < \lambda^2 \quad (6)$$

$$\bar{b} = \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}\hat{a}}^{-1} (\hat{a} - \bar{a}) \quad (7)$$

Fixed BIE

Reference: Odolinski, R., & Teunissen, P. J. G. (2020). Best integer equivariant estimation: Performance analysis using real data collected by low-cost, single- and dual- frequency, multi-GNSS receivers for short- to long-baseline RTK positioning. Journal of Geodesy, 94, 91.

Properties of the BIE

Part 1: RTK Google Pixel 4 and Samsung Galaxy S20, ILS and BIE.

Part 2: RTK Google Pixel5, Pixel6 Pro, Pixel7 Pro and Samsung Galaxy S22.

It has been shown in Teunissen (2003) that the BIE is unbiased and has minimum variance (dispersion D(.)), i.e.

$$
\begin{aligned}
&E\left(\bar{b}\right) = E\left(\check{b}\right) = E(\hat{b}) = b \\
&D\left(\bar{b}\right) \leq D\left(\check{b}\right) \qquad (8) \\
&D\left(\bar{b}\right) \leq D(\hat{b})
\end{aligned}
$$

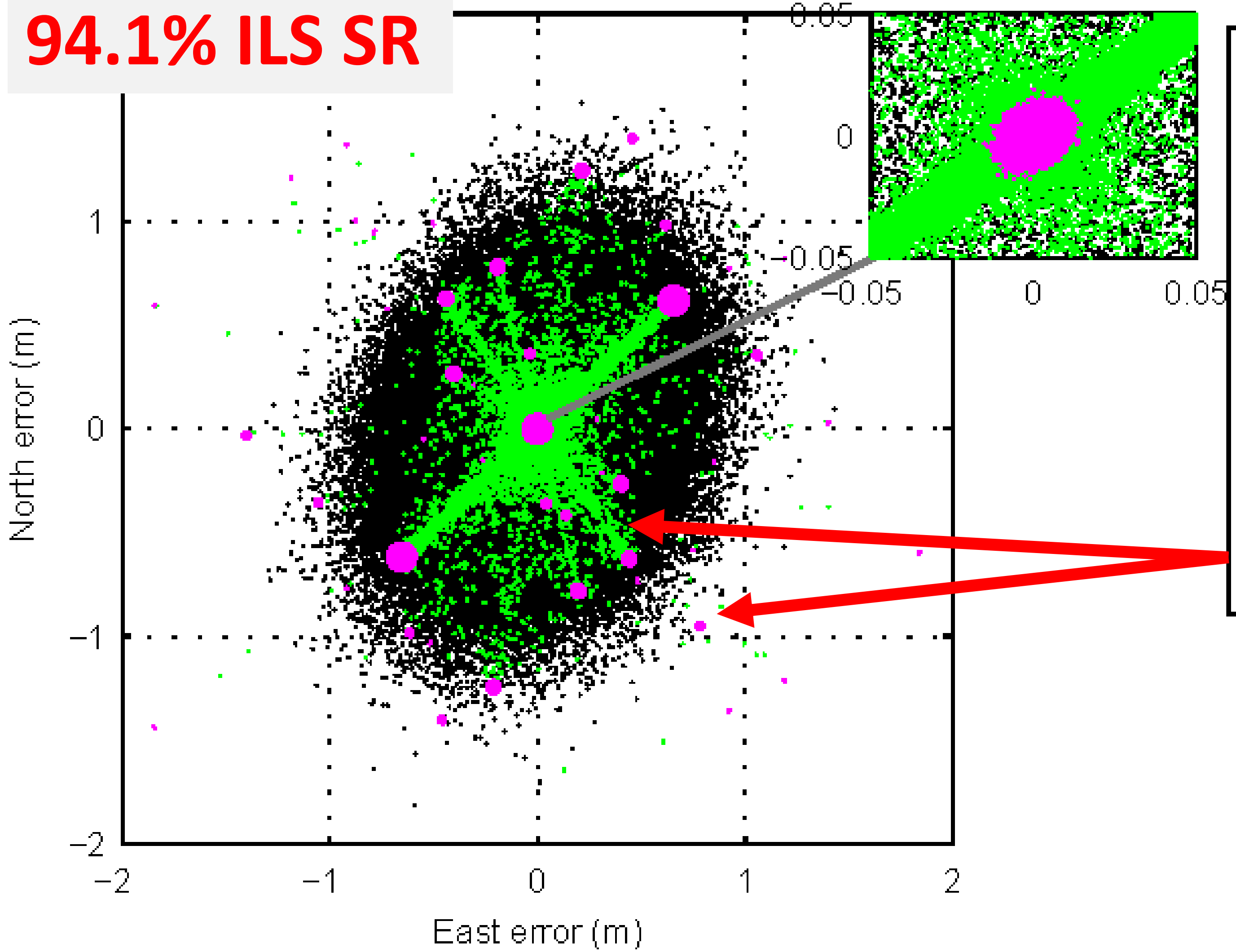
In other words precision of BIE ≤ ILS and BIE ≤ **Float**

BIE gives more reliable positions when the ILS success rate, i.e. the probability of correct integer ambiguity resolution, is different from one. It also avoids the need for the use of integer ambiguity validation techniques (like the fixed failure ratio test).

Reference: Odolinski, R., & Teunissen, P. J. G. (2020). Best integer equivariant estimation: Performance analysis using real data collected by low-cost, single- and dual- frequency, multi-GNSS receivers for short- to long-baseline RTK positioning. Journal of Geodesy, 94, 91.

Simulated single-epoch SF L1 GPS RTK positioning

94.1% ILS SR



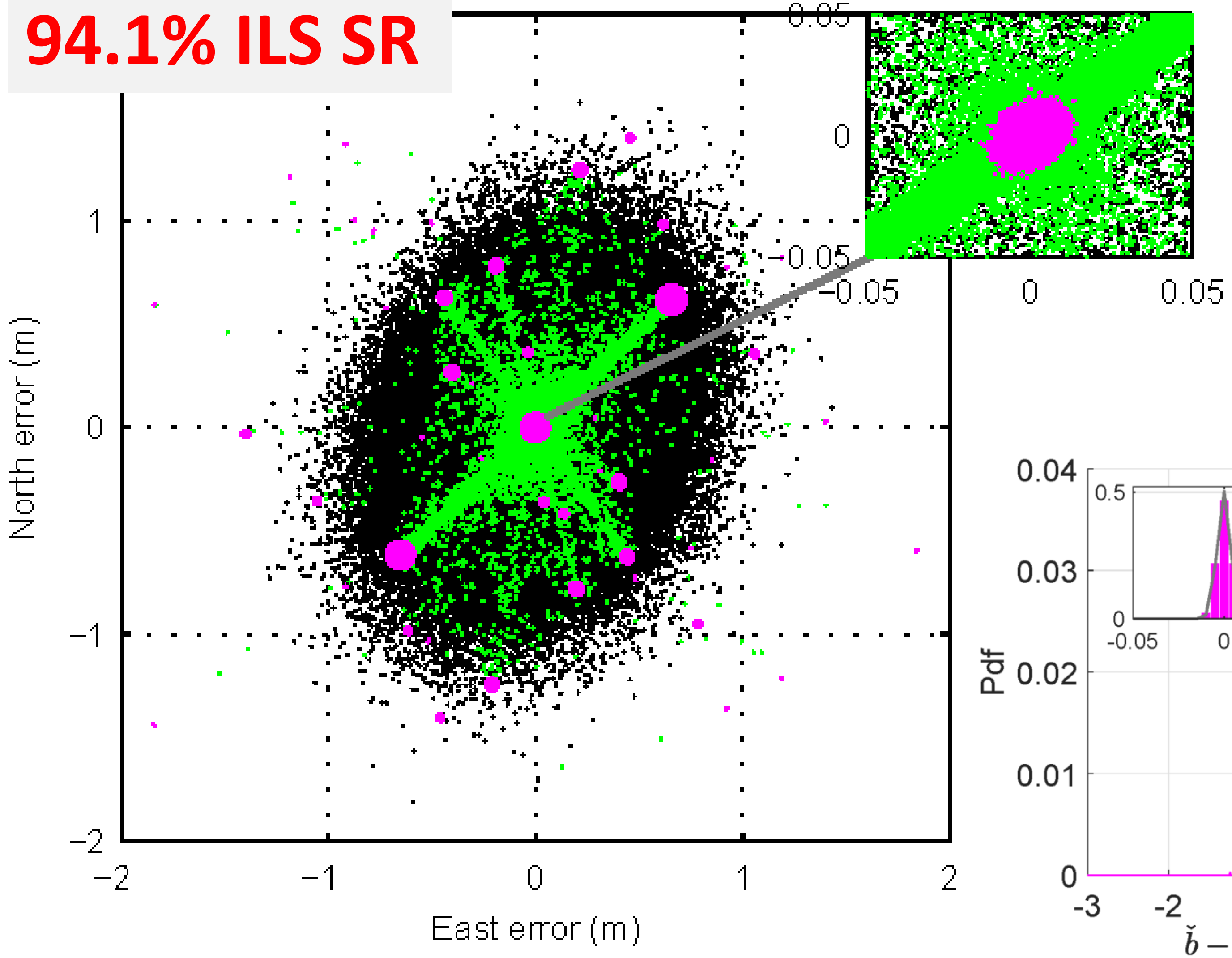
BIE follows a 'star-like' distributional pattern, and for instances when the distance between these ILS solutions and the **float** solutions are large, these incorrect ILS solutions are shown to not be as heavily weighted into the BIE solutions

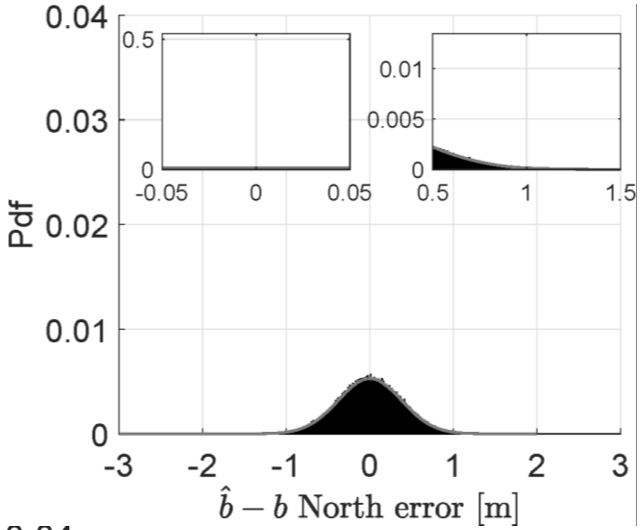
Float = black, ILS = magenta, BIE = green

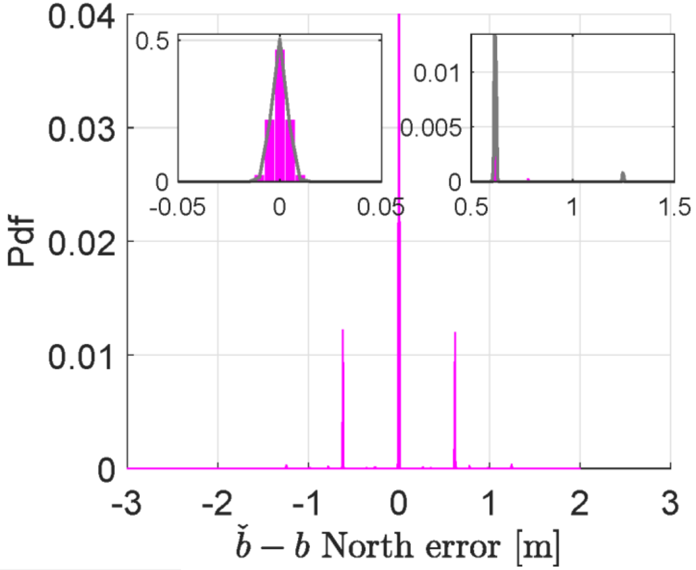
Reference: Odolinski, R., & Teunissen, P. J. G. (2020). Best integer equivariant estimation: Performance analysis using real data collected by low-cost, single- and dual- frequency, multi-GNSS receivers for short- to long-baseline RTK positioning. Journal of Geodesy, 94, 91.

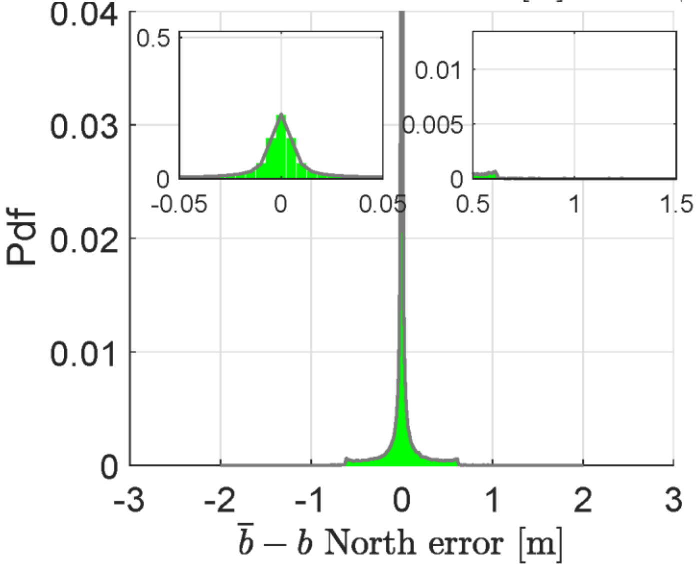
Simulated single-epoch SF L1 GPS RTK positioning

94.1% ILS SR









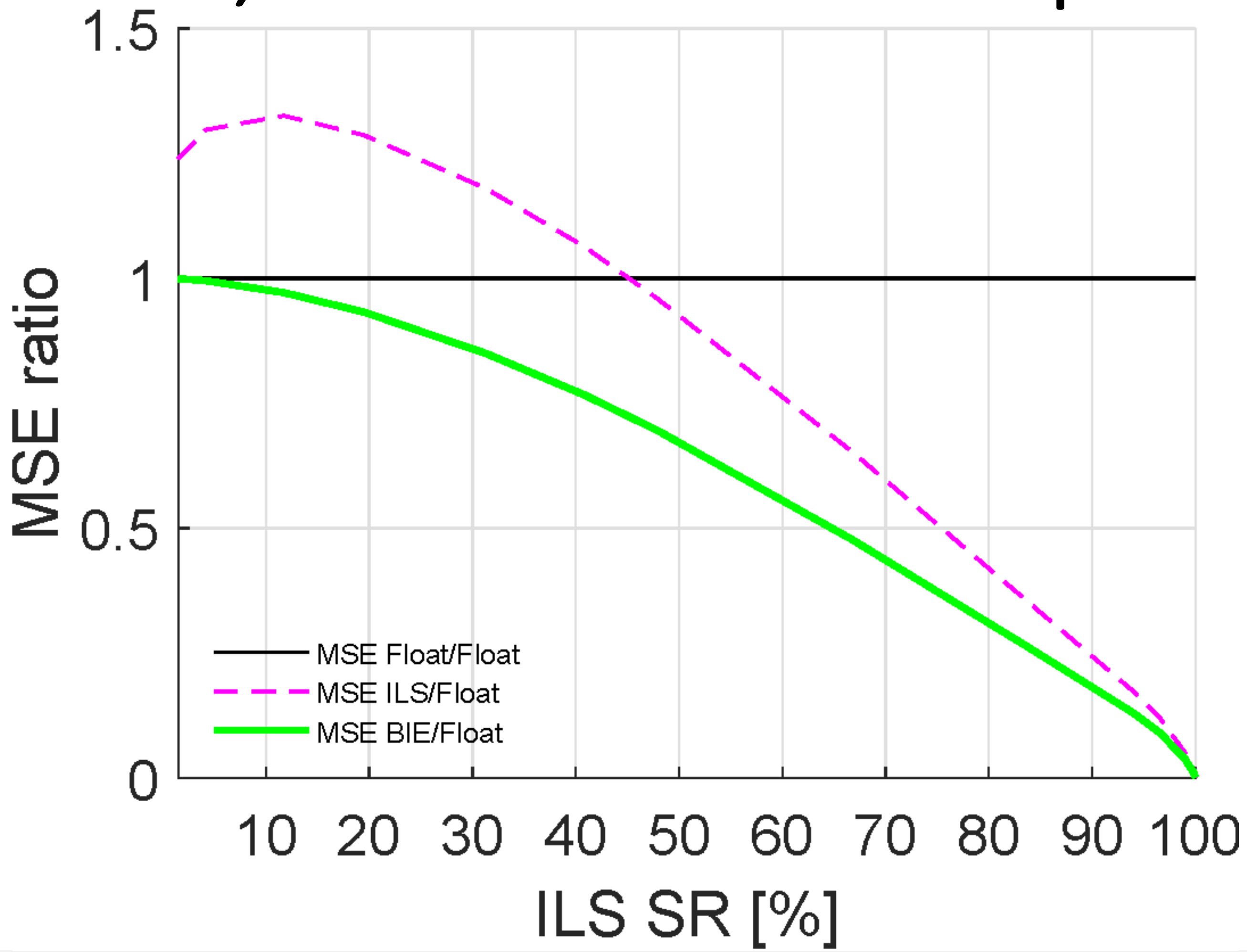
Float = black, ILS = magenta, BIE = green

Reference: Odolinski, R., & Teunissen, P. J. G. (2020). Best integer equivariant estimation: Performance analysis using real data collected by low-cost, single- and dual- frequency, multi-GNSS receivers for short- to long-baseline RTK positioning. Journal of Geodesy, 94, 91.

BIE, ILS and Float Mean Square Errors

Part 1: RTK Google Pixel 4 and Samsung Galaxy S20, ILS and BIE.

Part 2: RTK Google Pixel5, Pixel6 Pro, Pixel7 Pro and Samsung Galaxy S22.

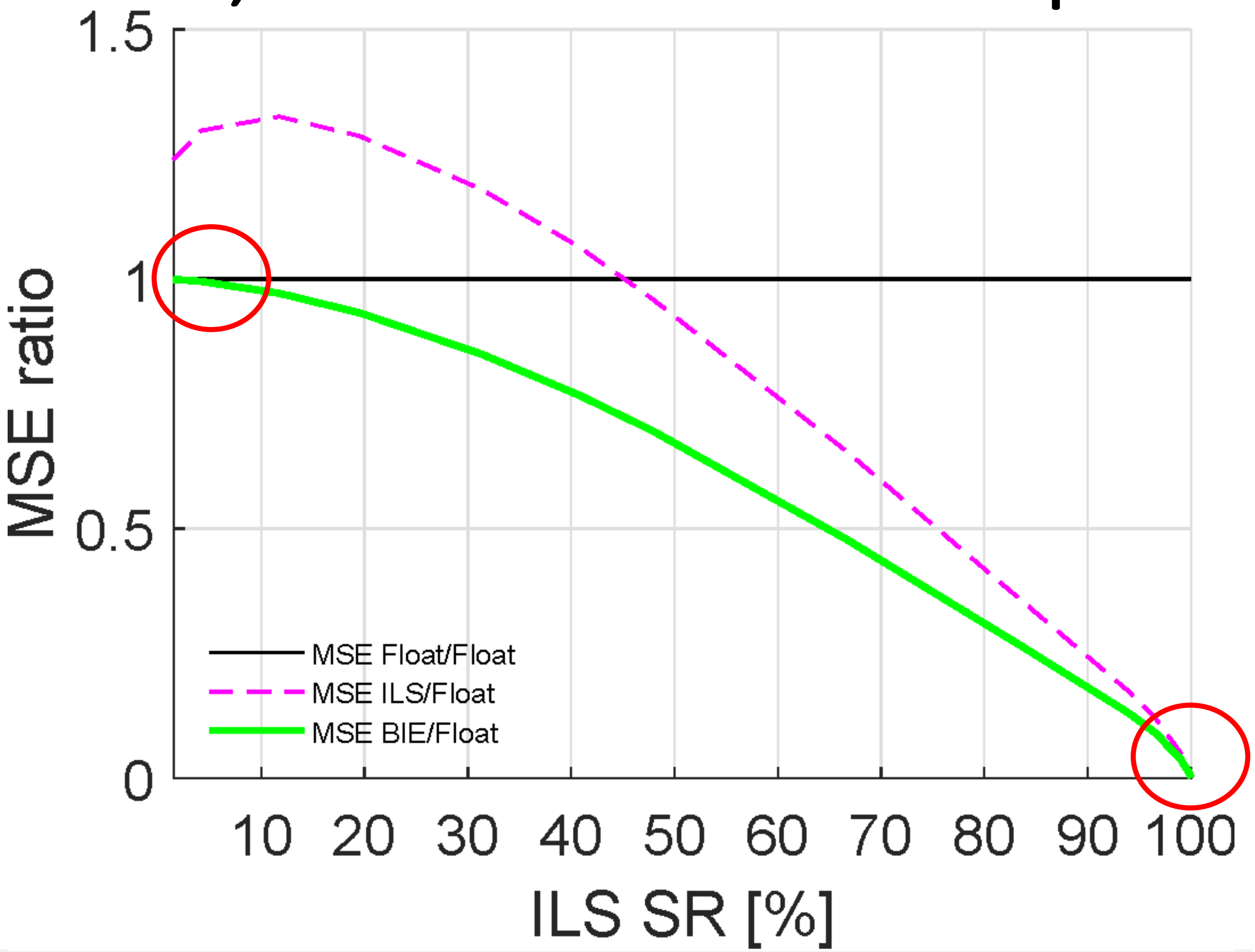


Mean Square Error (MSE) ratios (vs float) of

Float = black, ILS = magenta, BIE = green

Reference: Odolinski, R., & Teunissen, P. J. G. (2020). Best integer equivariant estimation: Performance analysis using real data collected by low-cost, single- and dual- frequency, multi-GNSS receivers for short- to long-baseline RTK positioning. Journal of Geodesy, 94, 91.

BIE, ILS and Float Mean Square Errors



Mean Square Error (MSE) ratios (vs float) of

Float = black, ILS = magenta, BIE = green

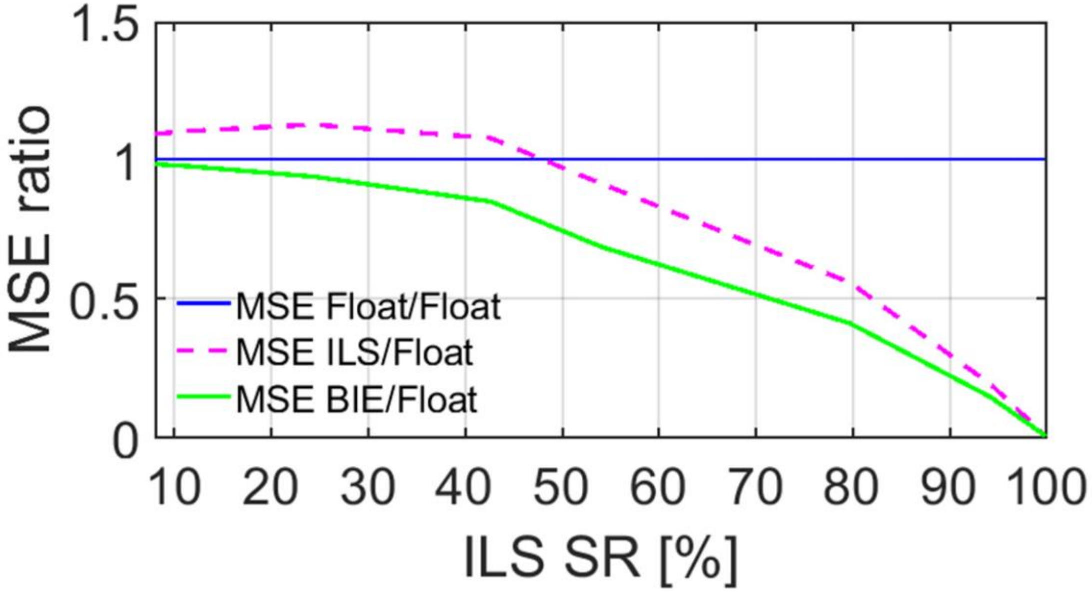
The MSEs of BIE are always smaller than both **float** and ILS, where in the limiting cases, with ILS SRs approaching 0% and 100%, we can see that BIE becomes equal to the **float** and ILS solutions, respectively

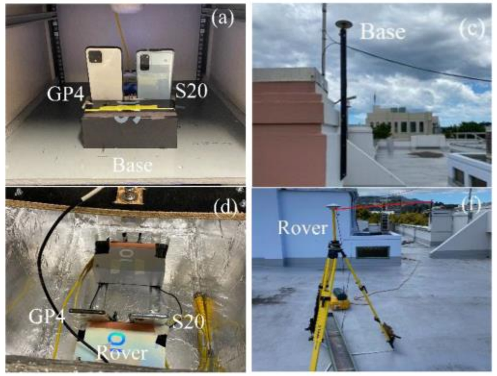
$$D\left(\bar{b}\right) \leq D\left(\check{b}\right)$$
$$D\left(\bar{b}\right) \leq D(\hat{b}) \quad (8)$$

Reference: Odolinski, R., & Teunissen, P. J. G. (2020). Best integer equivariant estimation: Performance analysis using real data collected by low-cost, single- and dual- frequency, multi-GNSS receivers for short- to long-baseline RTK positioning. Journal of Geodesy, 94, 91.

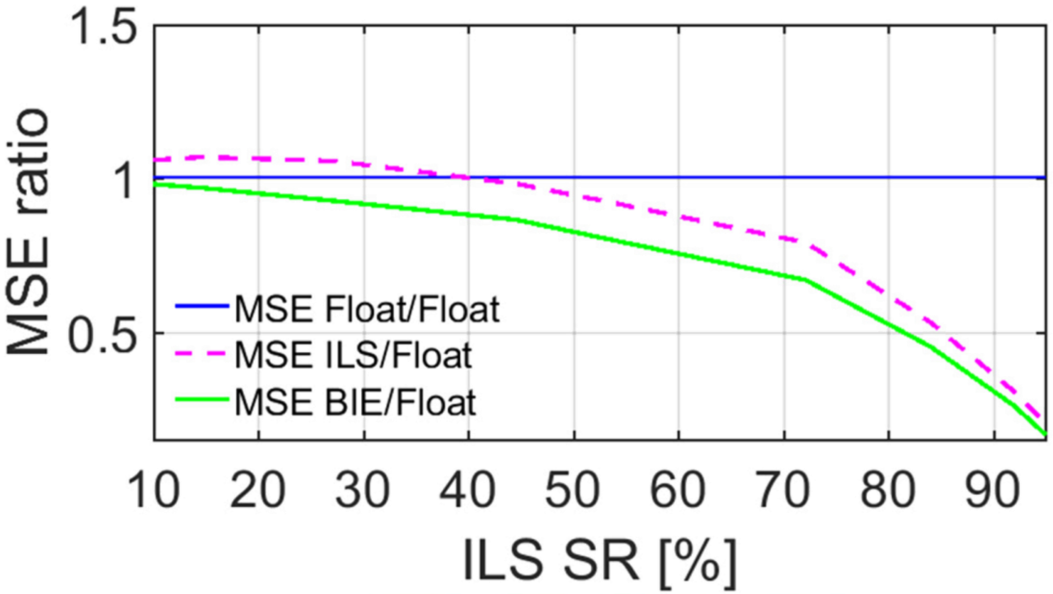
Mean Squared Errors (MSE) ratios for Google Pixel 4

External antennas SF RTK





Internal antennas DF RTK



Reference: Yong, C. Z., Harima, K., Rubinov, E., McClusky, S., & Odolinski, R. (2022). Instantaneous Best Integer Equivariant Position Estimation Using Google Pixel 4 Smartphones for Single-and Dual-Frequency, Multi-GNSS Short-Baseline RTK. Sensors, 22(10), 3772.

Conclusions

Part 1: RTK Google Pixel 4 and Samsung Galaxy S20, ILS and BIE.

Part 2: RTK Google Pixel5, Pixel6 Pro, Pixel7 Pro and Samsung Galaxy S22.

- When having the smartphones lying down, the RTK positioning performance, for S20 in particular, will deteriorate.
- We demonstrated, for the first time, a near hundred percent (98.7%) instantaneous RTK integer least-squares success rate for GP4 smartphones and cm level positioning precision while using short-baseline experiments with internal antennas.
- The BIE performance resembles that of the float estimator when the ILS SR is very low and was similar to that of the ILS when the ILS SR is very high.
- We demonstrated that BIE outperformed both the float and the ILS estimators even when on the basis of real GP4 smartphone data while using external and internal smartphone antennas.

Part 2: Evaluation of the multi-GNSS, dual-frequency RTK positioning performance for some recent Android smartphone models in a phone-to-phone setup

Robert Odolinski[1,2], **Hongzhou Yang**[1,3], **Li-Ta Hsu**[1,4], **Mohammed Khider**[1], **Guoyu (Michael) Fu**[1], **Damien Dusha**[1]

1 Google LLC, Mountain View, California, USA
2 National School of Surveying, University of Otago, Dunedin, New Zealand
3 Department of Geomatics Engineering, University of Calgary, Calgary, Canada
4 Department of Aeronautical and Aviation Engineering, Hong Kong Polytechnic University, Hong Kong

Reference: Odolinski, R., Yang, H., Hsu, L.-T., Khider, M., Fu, G. (M.), Dusha, D. (2024) Evaluation of the Multi-GNSS, Dual-Frequency RTK Positioning Performance for Recent Android Smartphone Models in a Phone-to-Phone Setup. In *Proceedings of the 2024 International Technical Meeting of The Institute of Navigation*, Long Beach, California, January 2024, pp. 42-53. https://doi.org/10.33012/2024.19575

Google

GNSS data collection

Part 1: RTK Google Pixel 4 and Samsung Galaxy S20, ILS and BIE.

Part 2: RTK Google Pixel5, Pixel6 Pro, Pixel7 Pro and Samsung Galaxy S22.

Fig. 1. GNSS data collection (21 Nov, 2023) in Mountain View, USA

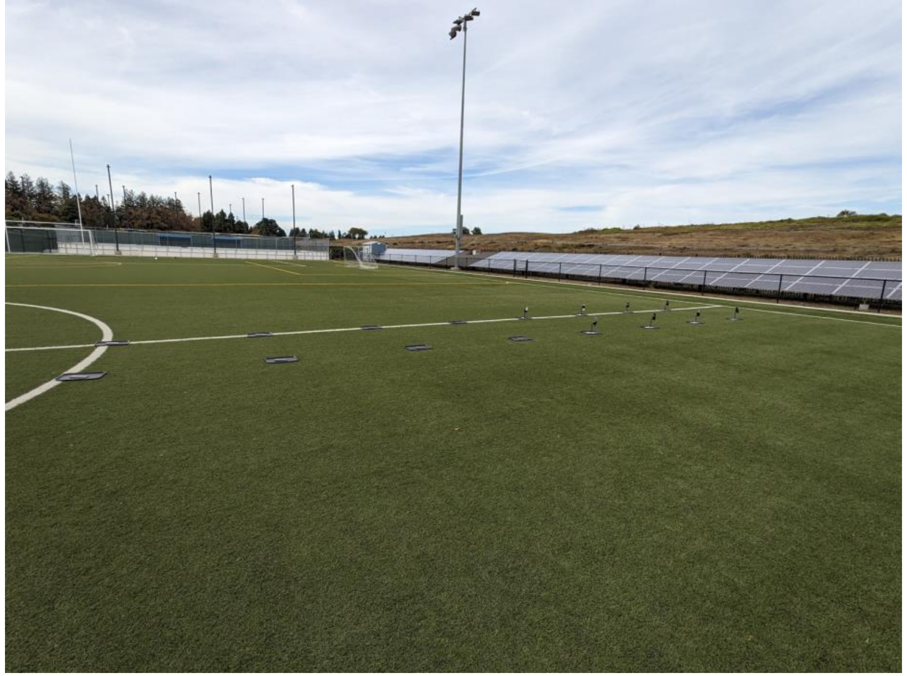


Table 1 2 h, 53 min and 48 s (1Hz) of short-baseline GNSS data using internal smartphone antennas (Fig. 1) and 5 degree cut-off angle

Android smartphone (GNSS chipset)	**GNSS signals tracked**
Google Pixel5 (Qualcomm Snapdragon 765G, SM7250)	**L1**,**L5**, **E1**, **E5a**, **B1I/B1c**, **L1**, **L5**, L1 GPS, Galileo, BDS, QZSS, GLONASS
Google Pixel6 Pro (Broadcom BCM47765)	**L1**,**L5**, **E1**, **E5a**, **B1I/B1c**, **B2a**, **L1**, **L5**, L1 GPS, Galileo, BDS, QZSS, GLONASS
Google Pixel7 Pro (Broadcom BCM47765)	**L1**,**L5**, **E1**, **E5a**, **B1I/B1c**, **B2a**, **L1**, **L5**, L1 GPS, Galileo, BDS, QZSS, GLONASS
Samsung S22 (Samsung SLSI K401)	**L1**,**L5**, **E1**, **E5a**, **B1I/B1c**, **B2a**, **L1**, **L5**, L1 GPS, Galileo, BDS, QZSS, GLONASS

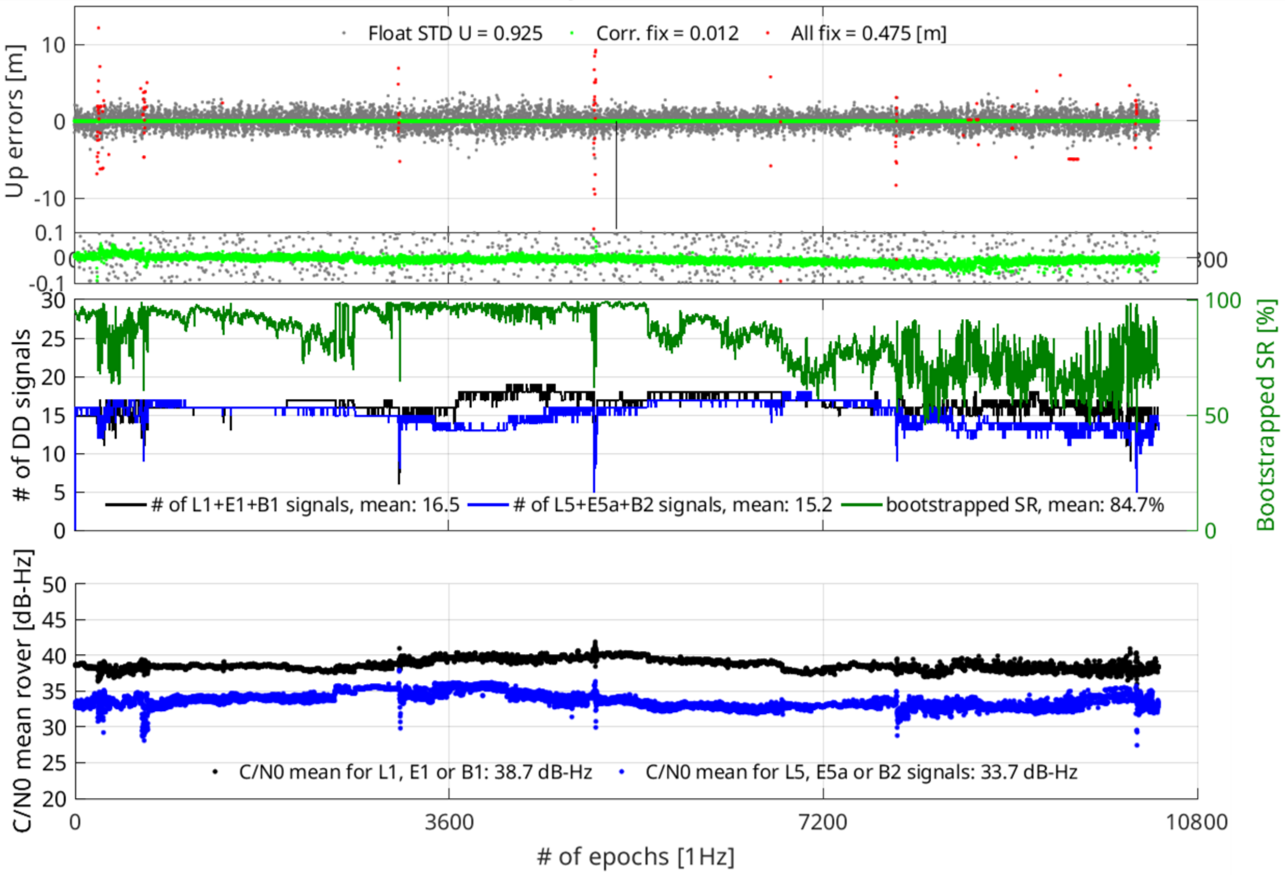
Reference: Odolinski, R., Yang, H., Hsu, L.-T., Khider, M., Fu, G. (M.), Dusha, D. (2024) Evaluation of the Multi-GNSS, Dual-Frequency RTK Positioning Performance for Recent Android Smartphone Models in a Phone-to-Phone Setup. In *Proceedings of the 2024 International Technical Meeting of The Institute of Navigation*, Long Beach, California, January 2024, pp. 42-53. https://doi.org/10.33012/2024.19575

Single-epoch RTK positioning with Android smartphones

Google Pixel7-to-Pixel7 RTK positioning (98.7% ILS SR):

L1+L5, E1+E5a, B1I+B2a GPS+Galileo+BDS

Outlier detection and C/N0 20 dB-Hz mask on



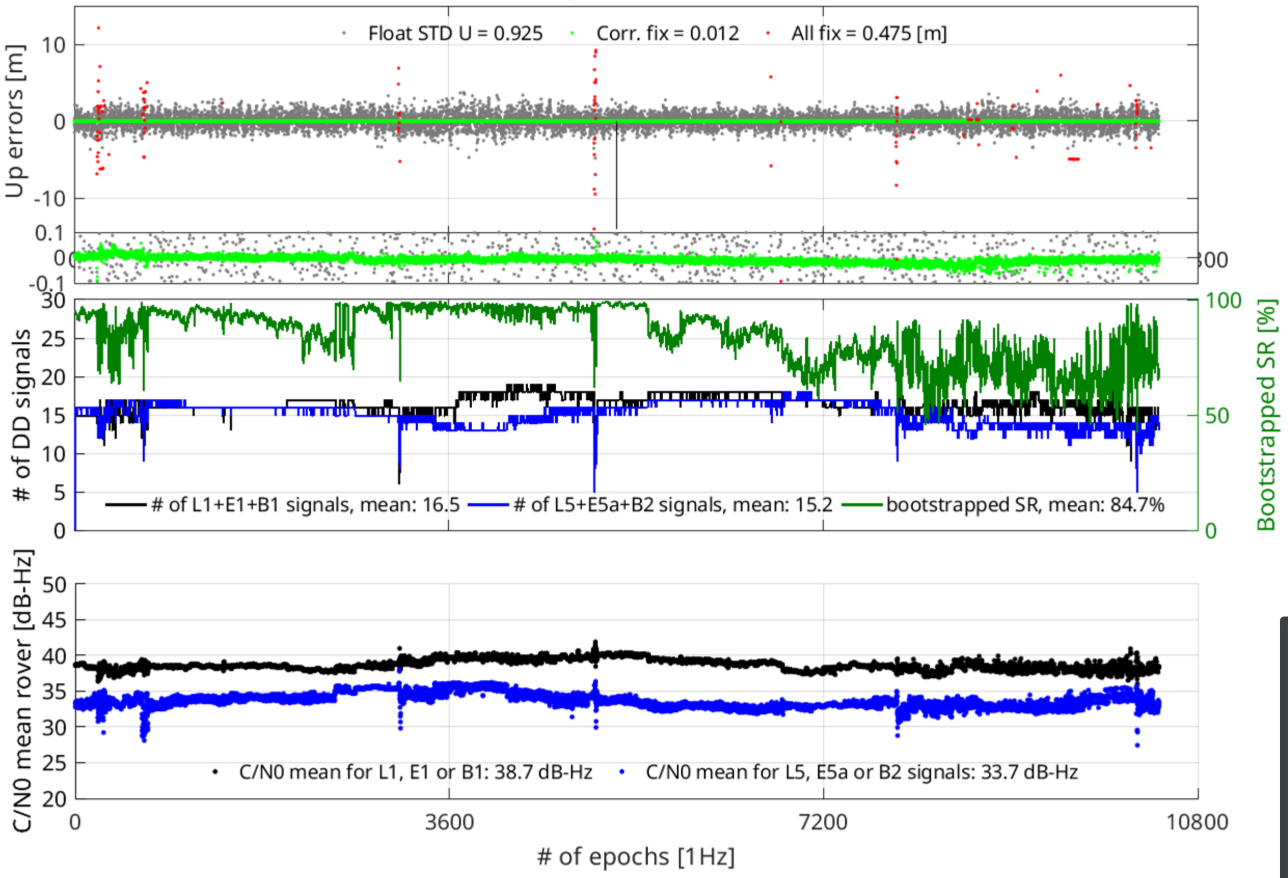
Reference: Odolinski, R., Yang, H., Hsu, L.-T., Khider, M., Fu, G. (M.), Dusha, D. (2024) Evaluation of the Multi-GNSS, Dual-Frequency RTK Positioning Performance for Recent Android Smartphone Models in a Phone-to-Phone Setup. In *Proceedings of the 2024 International Technical Meeting of The Institute of Navigation*, Long Beach, California, January 2024, pp. 42-53. https://doi.org/10.33012/2024.19575

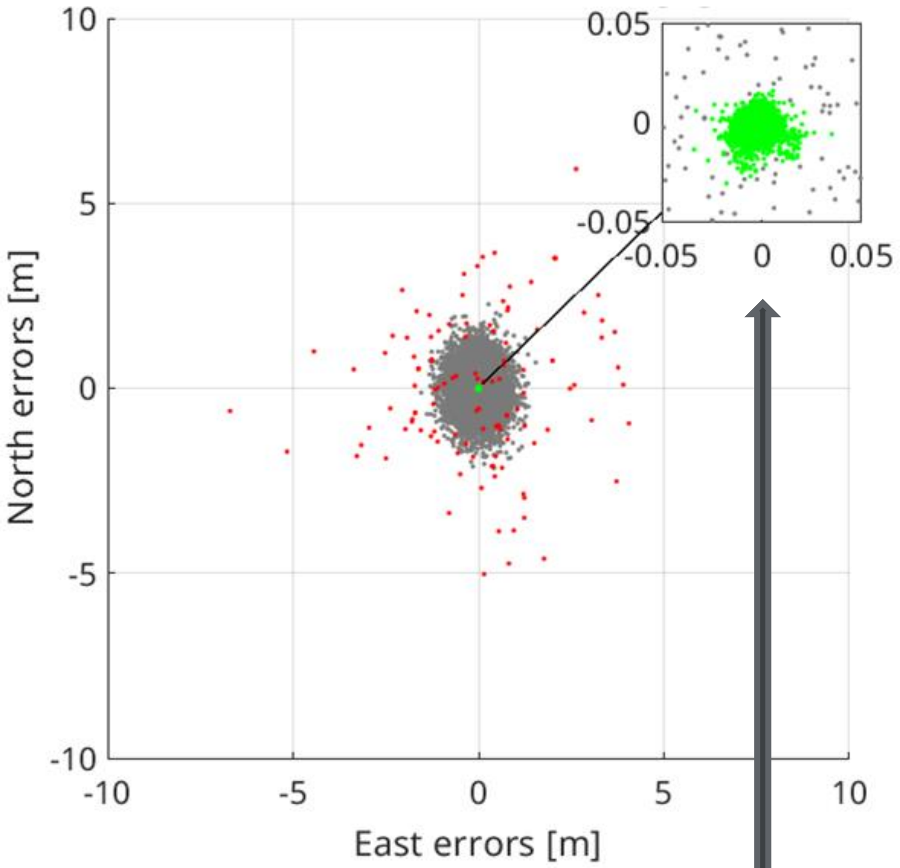
How about the single-epoch horizontal components?

Google Pixel7-to-Pixel7 RTK positioning (98.7% ILS SR):

L1+L5, E1+E5a, B1I+B2a GPS+Galileo+BDS

Outlier detection and C/N0 20 dB-Hz mask on





The horizontal North and East errors are shown with a zoom-in to also depict the **correctly fixed solutions at the mm-level**, and we can generally see a well distributed positioning error scatter plot.

Reference: Odolinski, R., Yang, H., Hsu, L.-T., Khider, M., Fu, G. (M.), Dusha, D. (2024) Evaluation of the Multi-GNSS, Dual-Frequency RTK Positioning Performance for Recent Android Smartphone Models in a Phone-to-Phone Setup. In *Proceedings of the 2024 International Technical Meeting of The Institute of Navigation*, Long Beach, California, January 2024, pp. 42-53. https://doi.org/10.33012/2024.19575

Single-epoch RTK positioning with Android smartphones

Android smartphone baseline (GNSS chipset)	GNSS signals tracked	ILS SR [%]	Corr. fix STD N/E/U [m]	Float STD N/E/U [m]
Samsung S22 to S22 (Samsung SLSI K401) ***GNSS data logging at start**	**L1**,**L5**, **E1**, **E5a**, **B1I/B1c**, **B2a** GPS, Galileo, BDS	**97.8***	**0.009** **0.005** **0.015**	**1.29** **0.78** **2.11**
Google Pixel5 to Pixel5 (Qualcomm Snapdragon 765G, SM7250) ****needs further investigation**	**L1**,**L5**, **E1**, **E5a**, **B1I/B1c** GPS, Galileo, BDS	**99.4**	**0.004** **0.017*** **0.007**	**0.59** **0.41** **1.07**
Google Pixel6 to Pixel6 (Pro) (Broadcom BCM47765)	**L1**,**L5**, **E1**, **E5a**, **B1I/B1c**, **B2a** GPS, Galileo, BDS	**98.3**	**0.004** **0.006** **0.017**	**0.70** **0.53** **1.24**
Google Pixel7 to Pixel7 (Pro) (Broadcom BCM47765)	**L1**,**L5**, **E1**, **E5a**, **B1I/B1c**, **B2a** GPS, Galileo, BDS	**98.7**	**0.004** **0.004** **0.012**	**0.54** **0.36** **0.93**

Note the > two order of magnitude improvement when going from the ambiguity-float to correctly fixed solutions

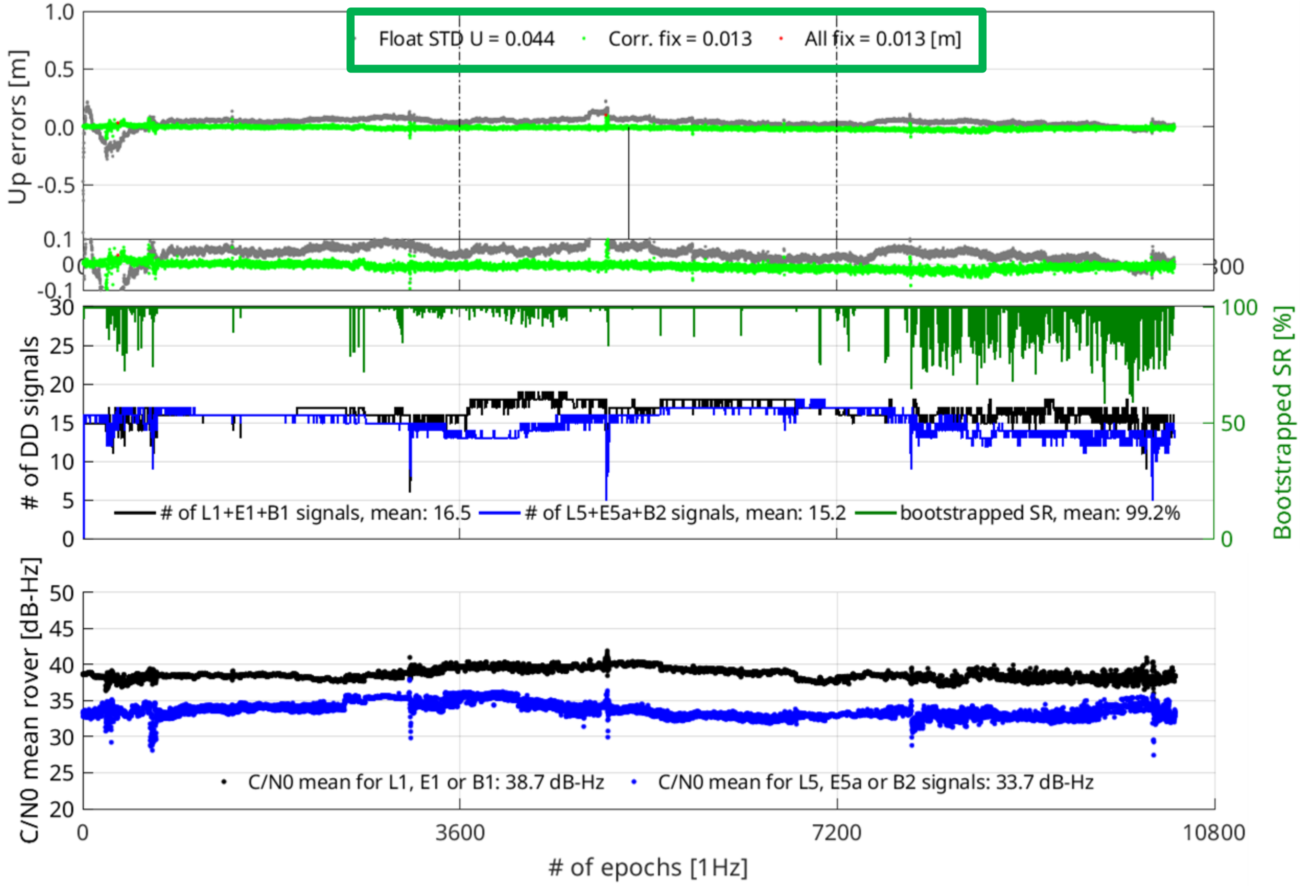
Reference: Odolinski, R., Yang, H., Hsu, L.-T., Khider, M., Fu, G. (M.), Dusha, D. (2024) Evaluation of the Multi-GNSS, Dual-Frequency RTK Positioning Performance for Recent Android Smartphone Models in a Phone-to-Phone Setup. In *Proceedings of the 2024 International Technical Meeting of The Institute of Navigation*, Long Beach, California, January 2024, pp. 42-53. https://doi.org/10.33012/2024.19575

Multi-epoch RTK positioning with Android smartphones

Google Pixel7-to-Pixel7 RTK positioning (100.0% ILS SR):

L1+L5, E1+E5a, B1I+B2a GPS+Galileo+BDS

Outlier detection and C/N0 20 dB-Hz mask on



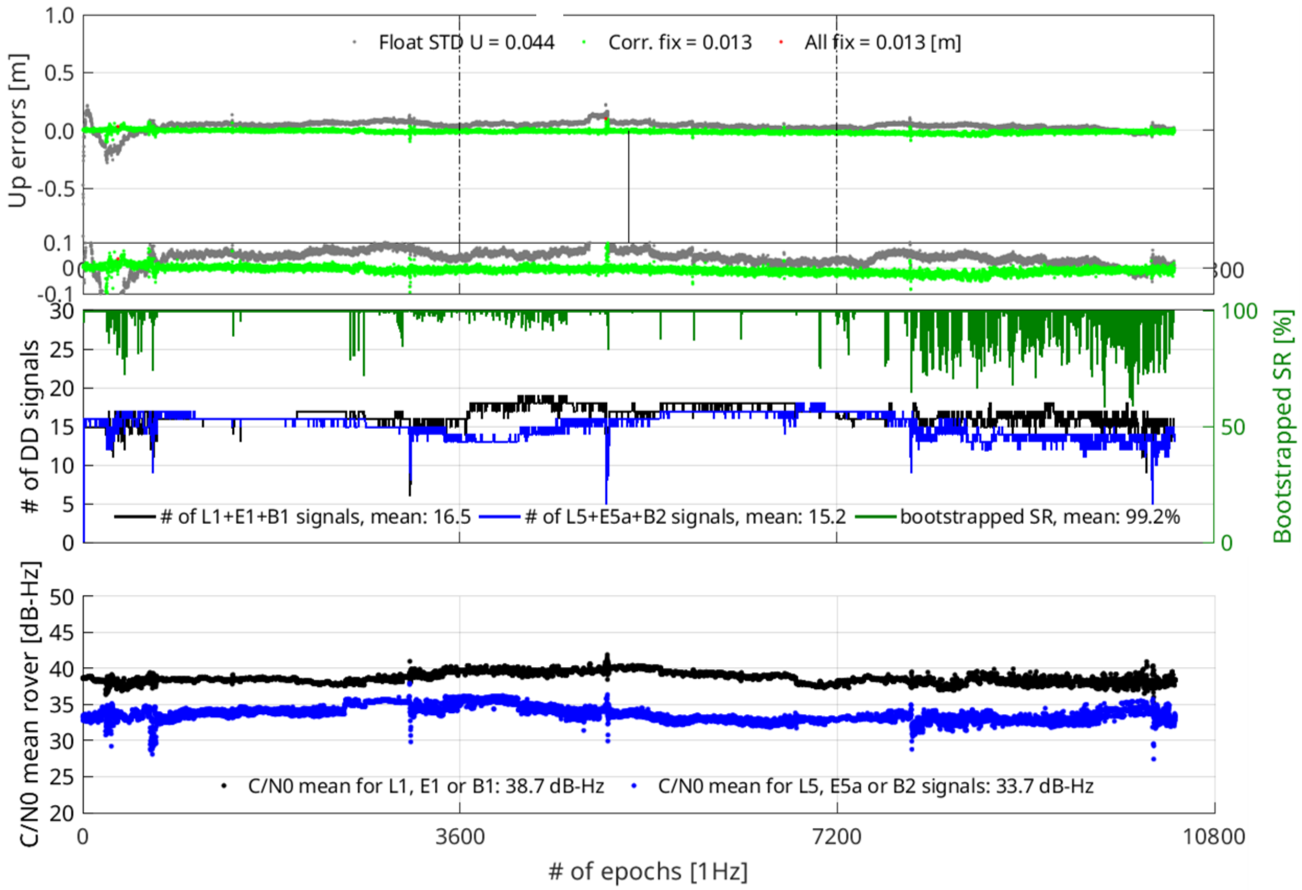
The **'multi-epoch'** model implies here that **the ambiguities are treated as time-constant parameters through a dynamic model** in a Kalman filter, and the precision of the ambiguity-float estimated positions is therefore expected to improve over time.

This is also shown by the **corresponding Up component STD** being 4.4 cm (compare to the **ambiguity-fixed Up STD** of 1.3 cm).

Reference: Odolinski, R., Yang, H., Hsu, L.-T., Khider, M., Fu, G. (M.), Dusha, D. (2024) Evaluation of the Multi-GNSS, Dual-Frequency RTK Positioning Performance for Recent Android Smartphone Models in a Phone-to-Phone Setup. In *Proceedings of the 2024 International Technical Meeting of The Institute of Navigation*, Long Beach, California, January 2024, pp. 42-53. https://doi.org/10.33012/2024.19575

Multi-epoch RTK positioning with Android smartphones

Google Pixel7-to-Pixel7 RTK positioning (100.0% ILS SR):
L1+L5, E1+E5a, B1I+B2a GPS+Galileo+BDS
Outlier detection and C/N0 20 dB-Hz mask on



The **'multi-epoch'** model implies here that **the ambiguities are treated as time-constant parameters through a dynamic model** in a Kalman filter, and the precision of the ambiguity-float estimated positions is therefore expected to improve over time.

This is also shown by the **corresponding Up component STD** being 4.4 cm (compare to the **ambiguity-fixed Up STD** of 1.3 cm).

Next we will **initialize the Kalman filter 3 times (every hour)**, which means we can compute a statistically relevant time to first fix (TTFF) of each solution. In the following it is defined as when the North, East and Up components are all below the 0.1 meter threshold and remains so for at least 5 minutes.

Reference: Odolinski, R., Yang, H., Hsu, L.-T., Khider, M., Fu, G. (M.), Dusha, D. (2024) Evaluation of the Multi-GNSS, Dual-Frequency RTK Positioning Performance for Recent Android Smartphone Models in a Phone-to-Phone Setup. In *Proceedings of the 2024 International Technical Meeting of The Institute of Navigation*, Long Beach, California, January 2024, pp. 42-53. https://doi.org/10.33012/2024.19575

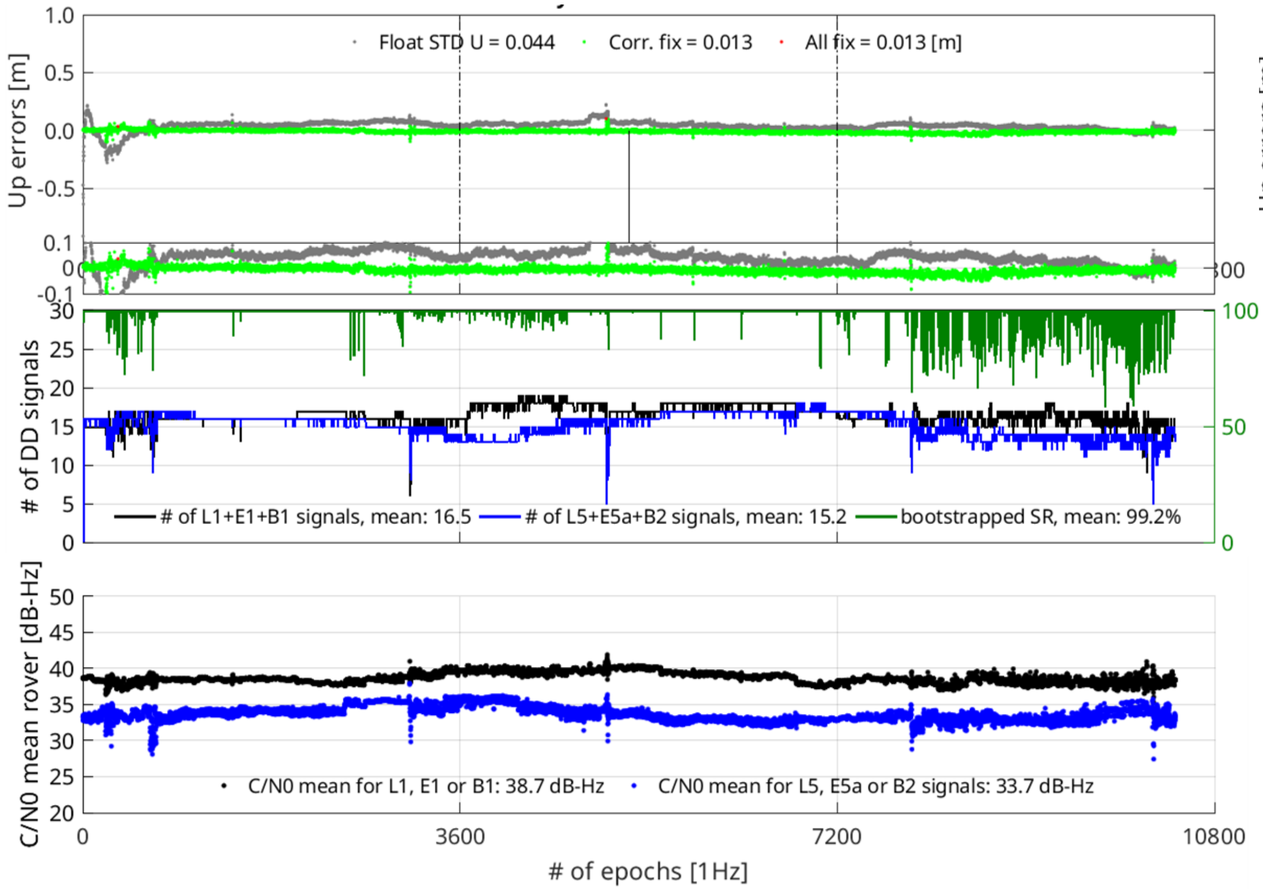
Multi-epoch RTK positioning with Android smartphones

Google Pixel7-to-Pixel7 RTK positioning

1 initialization **(100.0% ILS SR):** →

L1+L5, E1+E5a, B1I+B2a GPS+Galileo+BDS

Outlier detection and C/N0 20 dB-Hz mask on

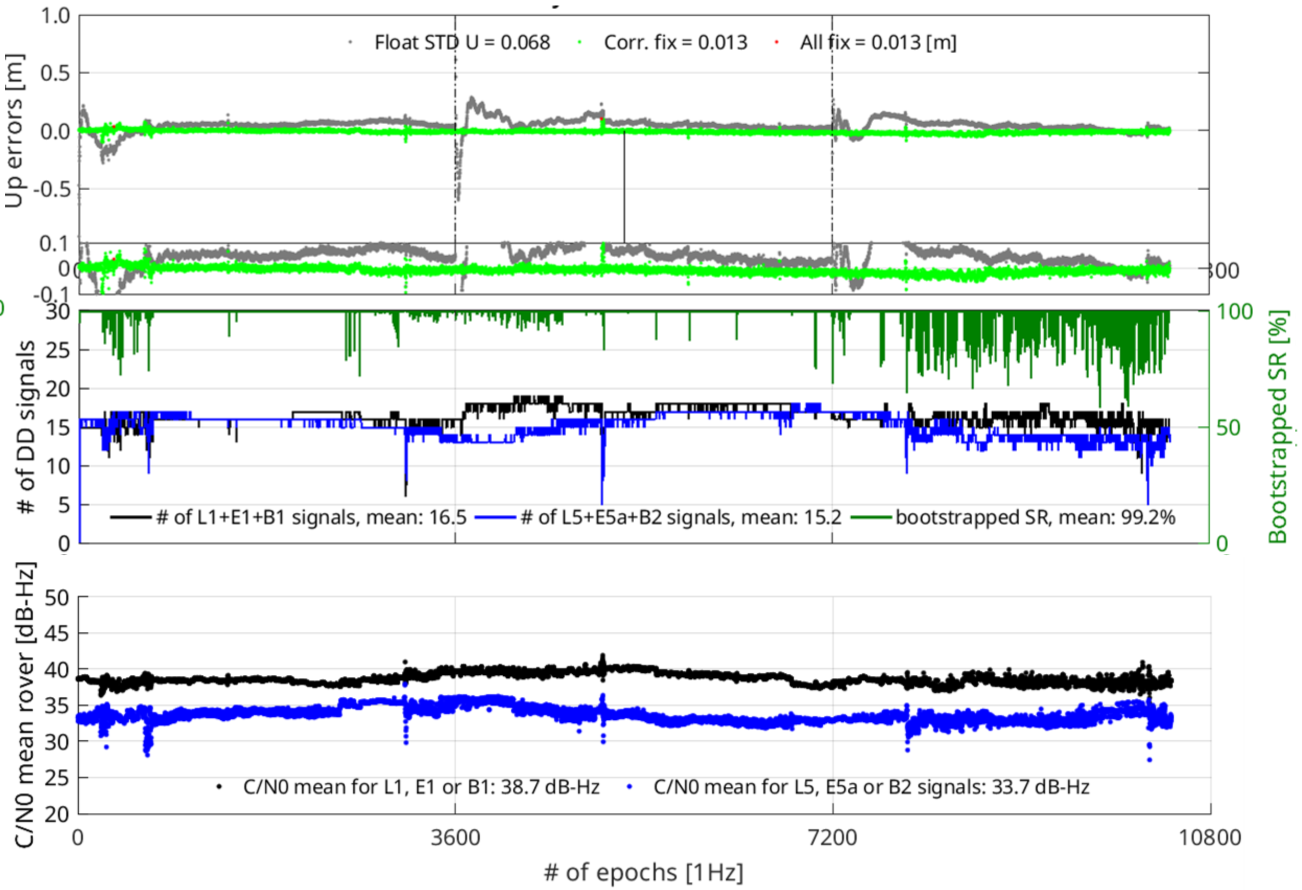


Google Pixel7-to-Pixel7 RTK positioning

3 re-initializations (100.0% ILS SR):

L1+L5, E1+E5a, B1I+B2a GPS+Galileo+BDS

Outlier detection and C/N0 20 dB-Hz mask on



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Multi-epoch RTK positioning with Android smartphones

Google Pixel7-to-Pixel7 RTK positioning

1 initialization **(100.0% ILS SR):** →

L1+L5, E1+E5a, B1I+B2a GPS+Galileo+BDS

Outlier detection and C/N0 20 dB-Hz mask on

Google Pixel7-to-Pixel7 RTK positioning

3 re-initializations (100.0% ILS SR):

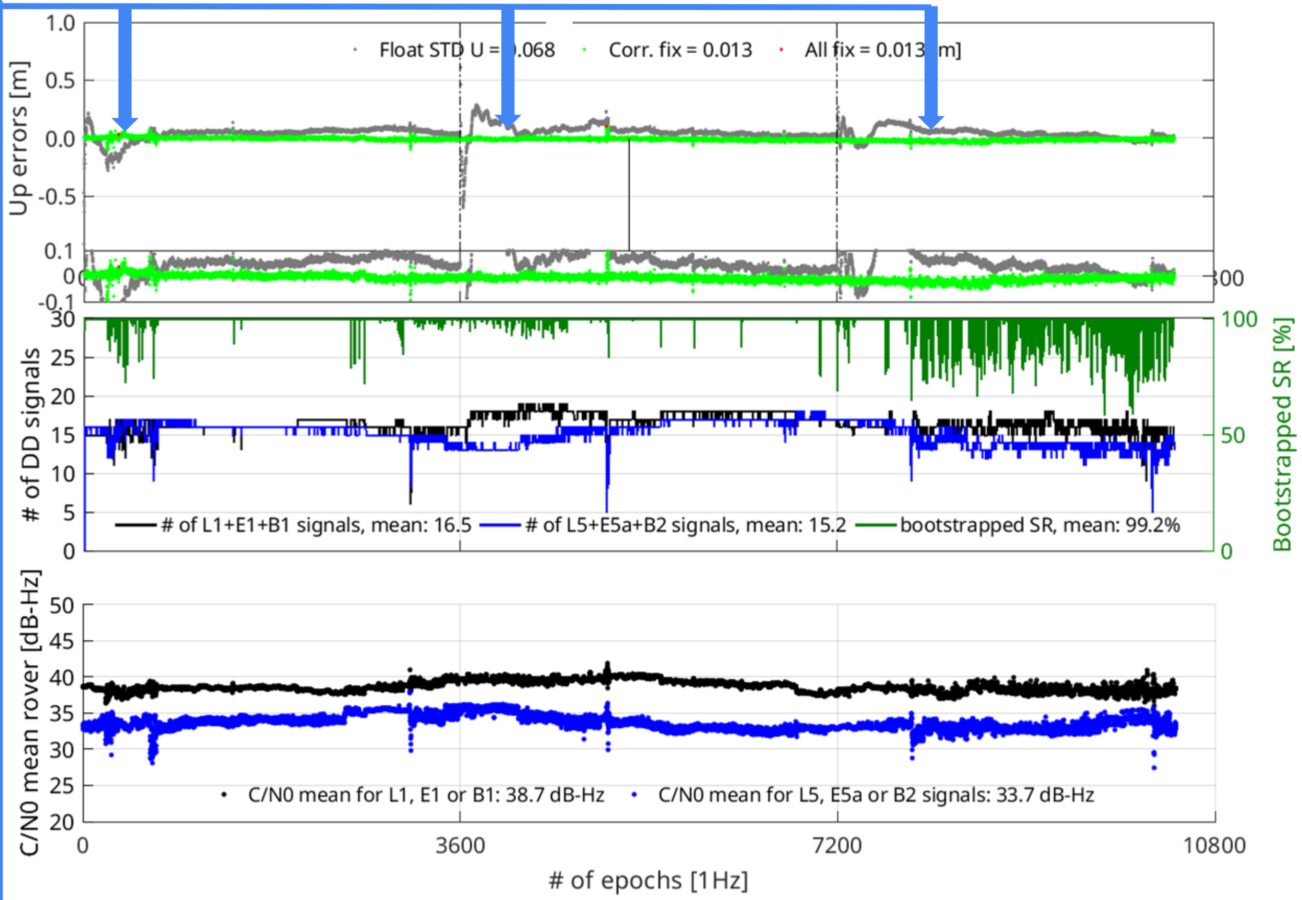
L1+L5, E1+E5a, B1I+B2a GPS+Galileo+BDS

Outlier detection and C/N0 20 dB-Hz mask on

The **TTFFs** to reach **< 0.1 m error** in North, East and Up for the **float solutions are:**

7 min 25 s,
8 min 26 s, and
12 min 16 s,

for the 1st, 2nd and 3rd initialization, respectively. This makes sense as the model strength is stronger for the first two initializations, respectively (see also the bootstrapped SR). The corresponding **fixed solutions all achieve such a threshold instantaneously**, i.e. within one epoch (although two of the solutions throughout the entire period become incorrectly fixed)

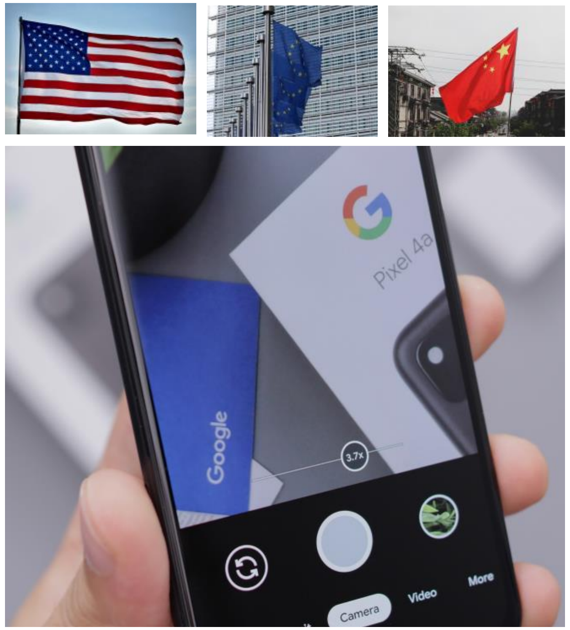


…uation of the Multi-GNSS, Dual-Frequency RTK Positioning Performance for Recent Android Smartphone Models in a Phone-to-Phone Setup. In *Proceedings of the 2024 International Technical Meeting of The Institute of Navigation*, Long Beach, California, January 2024, pp. 42-53. https://doi.org/10.33012/2024.19575

Conclusions

RTK tests were conducted with smartphones Google Pixel5, Pixel6 Pro, Pixel7 Pro, and Samsung Galaxy S22. Tracking dual frequency GPS, Galileo, QZSS, and BDS code and carrier phase observations.

- Achieved cm-level position and ~100% RTK integer least-squares success rates while stationary.



Source: all images from https://unsplash.com, used under https://unsplash.com/license

Reference: Odolinski, R., Yang, H., Hsu, L.-T., Khider, M., Fu, G. (M.), Dusha, D. (2024) Evaluation of the Multi-GNSS, Dual-Frequency RTK Positioning Performance for Recent Android Smartphone Models in a Phone-to-Phone Setup. In *Proceedings of the 2024 International Technical Meeting of The Institute of Navigation*, Long Beach, California, January 2024, pp. 42-53. https://doi.org/10.33012/2024.19575

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